University School of Automation and Robotics

GURU GOBIND SINGH INDRAPRASTHA UNIVERSITY

East Delhi Campus, Surajmal Vihar

Delhi - 110092

UNIVERSITY SCHOOL OF AUTOMATION AND ROBOTICS

Engineering Mechanics

By: Dr. Divya Agarwal

- **UNIT- I**

- **Force system:** Introduction, force, principle of transmissibility of force, resultant of a force system, resolution of a force, moment of force about a line, Varigon's theorem, couple, resolution of a force into force and a couple, properties of couple and their application to engineering problems.
- **Equilibrium:** Force body diagram, equations of equilibrium, and their applications to engineering problems, equilibrium of two force and three force members.
- **Distributed forces:** Determination of centre of gravity, centre of mass and centroid by direct integration and by the method of composite bodies., mass moment of inertia and area moment of inertia by direct integration and composite bodies method, radius of gyration, parallel axis theorem, polar moment of inertia.

- **UNIT- II**

- **Structure:** Plane truss, perfect and imperfect truss, assumption in the truss analysis, analysis of perfect plane trusses by the method of joints, method of section, graphical method.
- **Friction:** Static and Kinetic friction, laws of dry friction, co-efficient of friction, angle of friction, angle of repose, cone of friction, frictional lock, friction in pivot and collar bearing, friction in flat belts.

- **UNIT-III**

- **Kinematics of Particles:** Rectilinear motion, plane curvilinear motion, rectangular coordinates, normal and tangential coordinates
- **Kinetics of Particles:** Equation of motion, rectilinear motion and curvilinear motion, work energy equation, conservation of energy, concept of impulse and momentum, conservation of momentum, impact of bodies, co-efficient of restitution, loss of energy during impact.

- **UNIT-IV**

- **Kinematics of Rigid Bodies:** Concept of rigid body, type of rigid body motion, absolute motion, introduction to relative velocity, relative acceleration (Corioli's component excluded) and instantaneous center of zero velocity, velocity and acceleration.
- **Kinetics of Rigid Bodies:** Equation of motion, translatory motion and fixed axis rotation, application of work energy principles to rigid bodies conservation of energy.
- **Beam:** Introduction, types of loading, methods for the reactions of a beam, space diagram, types of end supports, beams subjected to couple

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DISTRIBUTED FORCES - INTRODUCTION

- Very often it is required to define a point such that the length of a wire, the area of a plate, the volume, the mass or the gravitational forces acting on a body may be assumed to be concentrated at that point.
- Such points are often called central points, such problems involves some distributed quantity.
- Earlier approach was to replace a distributed force by a single concentrated force – (Resultant method)
- Similar method shall be used now to determine the central points. Some commonly used central points are:

1. Centroid of the length of a curve
2. Centroid of the area of a surface
3. Centroid of the volume of a body
4. Mass centre of the mass of a body
5. Centre of gravity of the gravitational forces acting on a body.

DIFFERENCE BETWEEN CENTRE OF GRAVITY OF A BODY AND ITS CENTRE OF MASS

- **Centre of gravity of a body :** It is a point through which the resultant of the distributed gravity forces acts irrespective of the orientation of the body.

- **Centre of mass**: It is the point where the entire mass of a body may be assumed to be concentrated.

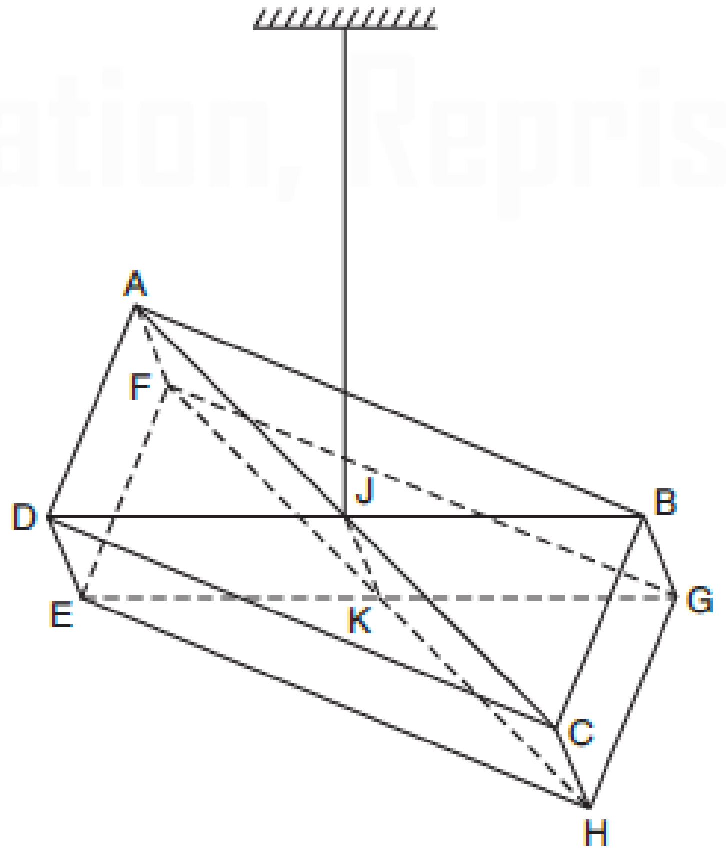
- The centre of mass and the centre of gravity of a body are different only when the gravitational field is not uniform and parallel.
- For practical purposes they are assumed to be the same.

CENTRE OF GRAVITY OF A BODY

- A body comprises of several parts and its every part possesses weight.
- *Weight is the force of attraction between a body and the earth and is proportional to mass of the body.*
- The weights of all parts of a body can be considered as parallel forces directed towards the centre of the earth.
- Therefore, they may be combined into a resultant force whose magnitude is equal to their algebraic sum.
- If a supporting force, equal and opposite to the resultant, is applied to the body along the line of action of the resultant, the body will be in equilibrium. This line of action will pass through the centre of gravity of the body.
- Thus, centre of gravity of the body may be defined as the *point through which the whole weight of a body may be assumed to act.*
- The centre of gravity of a body or an object is usually denoted by c.g. or simply by *G*.
- The position of c.g. depends upon shape of the body and this may or may not necessarily be within the boundary of the body.

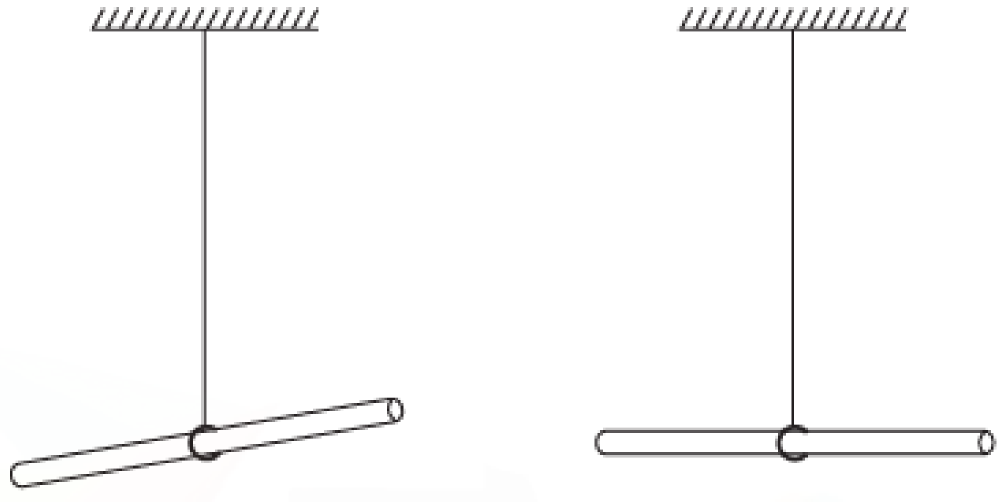
DETERMINATION OF CENTRE OF GRAVITY OF A BODY

- The centre of gravity of some objects may be found by *balancing the object on a point.*
- Take a thin plate of thickness *t*, shown in Figure.
- Draw the diagonals of the upper and lower forces to intersect at *J*, and *K* respectively.
- If the plate is placed on point at *K*, the plate will not fall i.e., it is balanced.
- If suspended from *J*, the plate will hang horizontally.
- The centre of gravity of the plate is at the centre of the line *JK*.



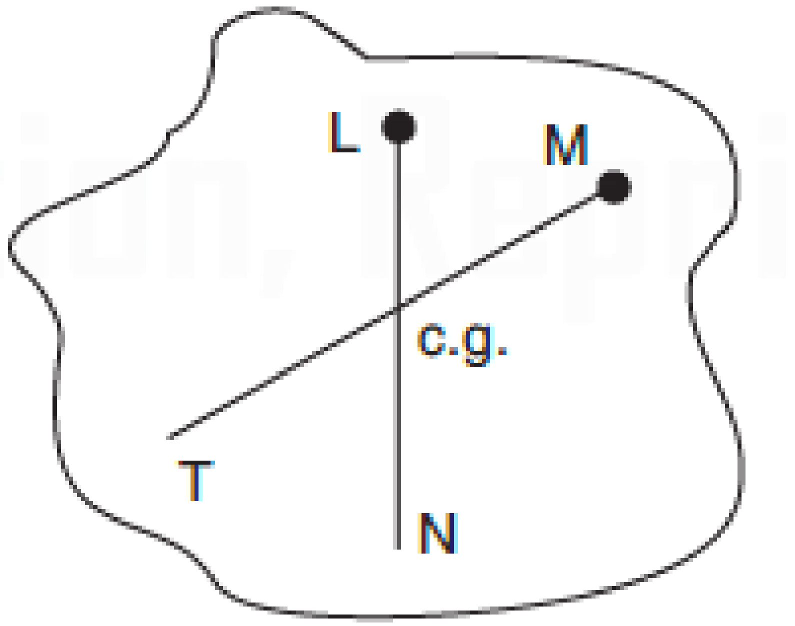
DETERMINATION OF CENTRE OF GRAVITY OF A BODY

- If we suspend a uniform rod by a string and move the position of the string until the rod hangs vertically, we can determine that the centre of gravity of the rod lies at its centre.
- Through the use of similar procedures it can be established that a body which has an axis, or line, symmetry has its centre of gravity located on that line, or axis.
- Of course, if a body has more than one axis of symmetry, the centre of gravity must lie at the intersection of the axes.



DETERMINATION OF CENTRE OF GRAVITY OF A BODY

- **Another method** for determining the centre of gravity is by *suspension*.
- Take an object, the section of which is shown in Figure.
- Suspend it from point *L*.
- The body will not come to rest until its resultant weight is vertically downward from *A*.
- Through *L*, draw a vertical line *LN*.
- Then suspend the body from a point *M*, and let come to rest.
- Through *M*, draw a vertical line *MT*.
- The point of *LN* and *MT* is the position of the centre of intersection of gravity.



CENTRE OF GRAVITY OF A BODY: DETERMINATION BY THE METHOD OF MOMENTS

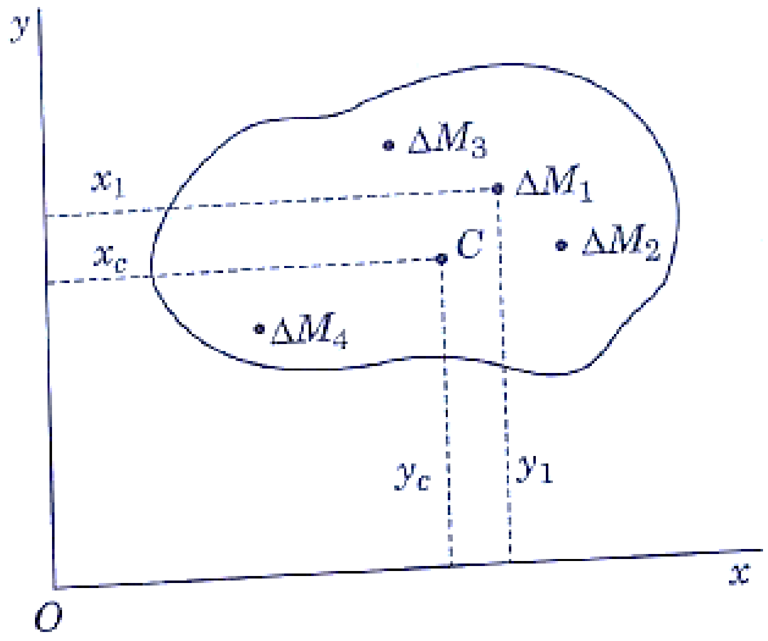
- Consider a body of mass M.
- Let this body be composed of a number of masses $\Delta \mathbf{M}_1, \Delta \mathbf{M}_2, \Delta \mathbf{M}_3, \ldots. \Delta \mathbf{M}_n$ distributed within the body such that

$$\mathbf{M} = \Delta \mathbf{M}_1 + \Delta \mathbf{M}_2 + \Delta \mathbf{M}_3 + \ldots. \Delta \mathbf{M}_n$$

- The distance of these masses w.r.t. the axis be,

$$(x_1, y_1), (x_2, y_2), \ldots\ldots. (x_n, y_n)$$

- Mathematically, it can be stated that the mass M of the body has been divided into 'n' elements of masses.
- Let centre of gravity of whole mass M lie at a distance (x_c, y_c) w.r.t. reference axes.
- Let us assume that the gravitational field is uniform and parallel.
- Gravitational force acting on the mass $\Delta \mathbf{M}_1 = \Delta \mathbf{M}_1 \mathbf{g}$. Similarly, we can find the gravitational forces acting on the masses $\Delta \mathbf{M}_2, \Delta \mathbf{M}_3, \ldots. \Delta \mathbf{M}_n$.



DETERMINATION OF CENTRE OF GRAVITY OF A BODY

- To find the resultant of parallel forces $\Delta M_1 g, \Delta M_2 g, \Delta M_3 g, \ldots \Delta M_n g$ we apply the principle of moments.
- Moment of the resultant of all forces about y-axis = Sum of moments of all the forces about y axis

$$Mg(x_c) = \Delta M_1 g(x_1) + \Delta M_2 g(x_2) + \Delta M_3 g(x_3) + \ldots \Delta M_n g(x_n)$$

$$x_c = \frac{\Delta M_1(x_1) + \Delta M_2(x_2) + \Delta M_3(x_3) + \ldots \Delta M_n(x_n)}{M}$$

where, $$M = \Delta M_1 + \Delta M_2 + \Delta M_3 + \ldots \Delta M_n$$

- Above equation can be expressed as $M = \sum(\Delta M_i)$
- The numerator can also be expressed in a similar manner. Therefore,

$$x_c = \frac{\sum \Delta M_i(x_i)}{\sum(\Delta M)_i} \quad \text{and} \quad y_c = \frac{\sum \Delta \mathrm{Mi}(\mathrm{y_i})}{\sum(\Delta M)_i}$$

- The summation sign Σ means that all the elements of the masses are to be considered.

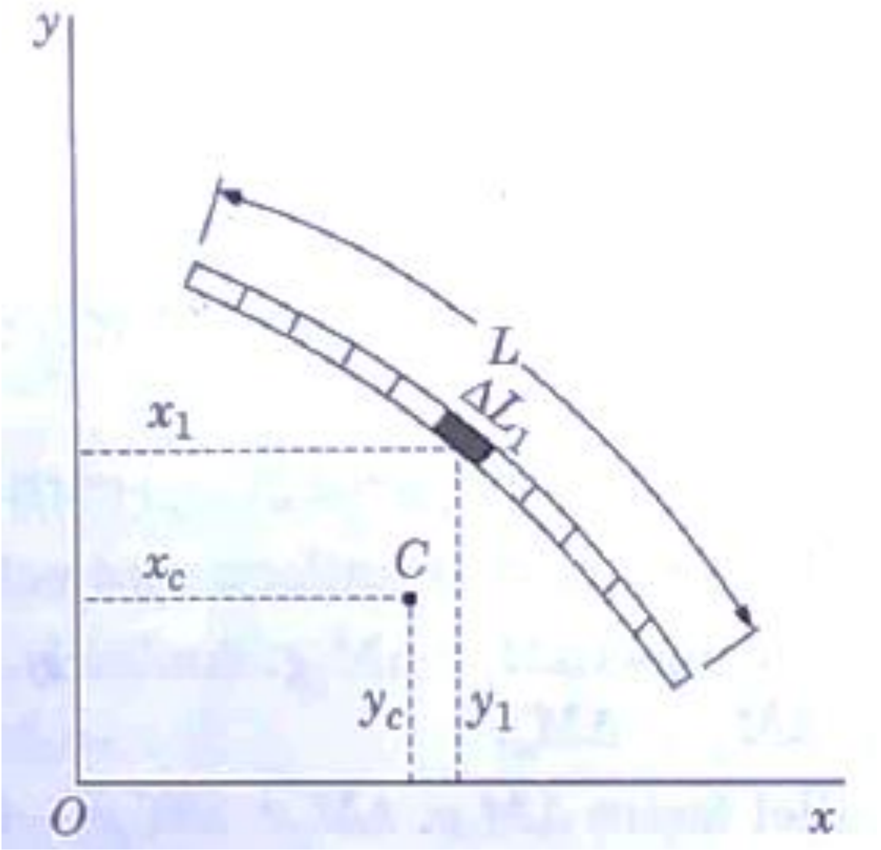
CENTROID

- *The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.*
- Thus, centroid can be taken as quite analogous to centre of gravity when bodies have area only and not weight.

CONCEPT OF CENTROID

- **One dimensional body (line segment).** Consider a body in the shape of a curved homogeneous wire of uniform cross section and of length L.
- Divide the length of the wire into elements of lengths $\Delta \mathbf{L_1}$, $\Delta \mathbf{L_2}$,...... $\Delta \mathbf{L_n}$.
- Let the uniform area of cross section=A
- Density of the wire = ρ
- The mass M of the wire of length L = ALρ
- The mass M of an element of length $\Delta L_1 = \Delta M_1$

  $$\Delta M_1 = \text{volume} * \text{density} = A\,(\Delta L_1)\rho$$
- Similarly the masses $\Delta \mathbf{M_2}$, $\Delta \mathbf{M_3}$, …. $\Delta \mathbf{M_n}$ of other elements can be determined.



CONCEPT OF CENTROID

- Let the distances of the centres of these lengths w.r.t. the axis be, $(x_1,y_1), (x_2,y_2), \ldots\ldots (x_n, y_n)$
- Applying the principle of moments,

$$x_c = \frac{\sum \Delta M_i(x_i)}{\sum(\Delta M)_i}$$

$$x_c = \frac{A(\Delta L_1)\rho(x_1)+A(\Delta L_2)\rho(x_2)+A(\Delta L_3)\rho(x_3)+ \ldots A(\Delta L_n)\rho(x_n)}{A(\Delta L_1)\rho+A(\Delta L_2)\rho+A(\Delta L_3)\rho+ \ldots A(\Delta L_n)\rho}$$

$$x_c = \frac{A\rho(\Delta L_1x_1+\Delta L_2x_2+\Delta L_3x_3+ \ldots \Delta L_nx_n)}{A\rho(\Delta L_1+\Delta L_2+\Delta L_3+ \ldots \Delta L_n)} = \frac{\sum \Delta \mathrm{L}_i(x_i)}{\sum(\Delta L)_i}$$

- Similarly, $y_c = \dfrac{\sum \Delta \mathrm{L}_i(y_i)}{\sum(\Delta L)_i}$
- Because density ρ and area of cross section A are constant over entire length of the wire, the coordinates of centre of gravity of the wire becomes the coordinates of the centroid of the wire generally called the centroid of a line segment.

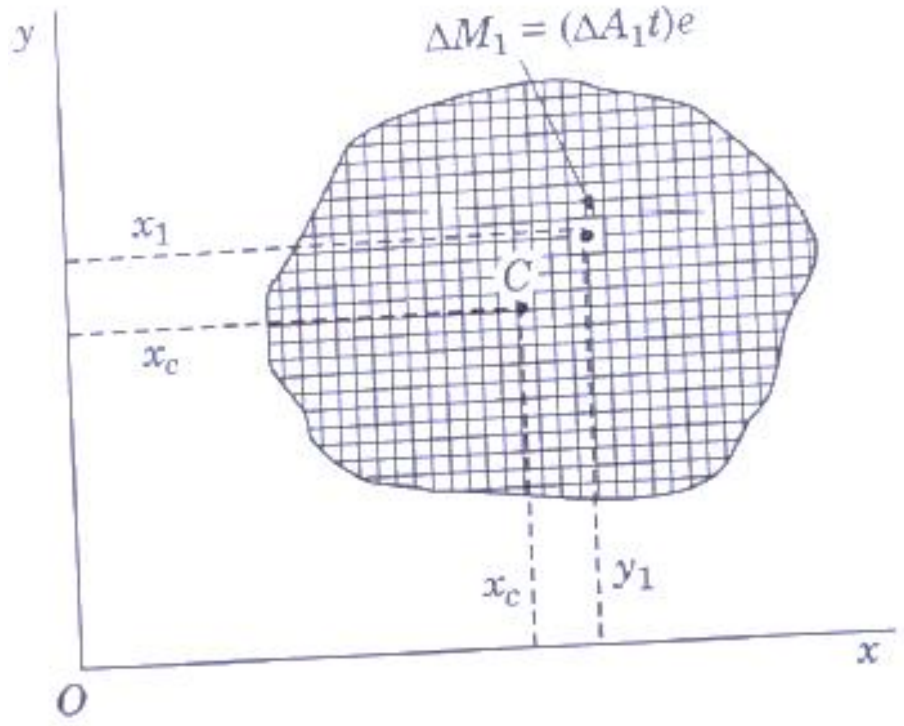
CENTROID TWO DIMENSIONAL BODY

- Now consider the case of a homogeneous plate or lamina of uniform thickness t and density **ρ** and total area A.
- Divide area of plate into elements of areas $\Delta A_1, \Delta A_2, \Delta A_3, \ldots \Delta A_n$.
- The distances of the centres of these areas w.r.t. the axis be (x_1,y_1), (x_2,y_2), ……. (x_n, y_n)
- The mass of the plate, $M = At\rho$
- Mass ΔM_1 of the element, $\Delta M_1 = \Delta A_1 t\rho$
- Using,

$$x_c = \frac{\sum \Delta M_i(x_i)}{\sum(\Delta M)_i}$$

$$x_c = \frac{\Delta A_1 t\rho(x_1)+\Delta A_2 t\rho(x_2)+\Delta A_3 t\rho(x_3)+ \ldots\Delta A_n t\rho(x_n)}{\Delta A_1 t\rho+\Delta A_2 t\rho+\Delta A_3 t\rho+ \ldots\Delta A_n t\rho}$$

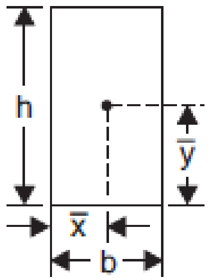
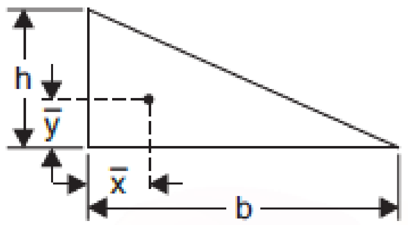
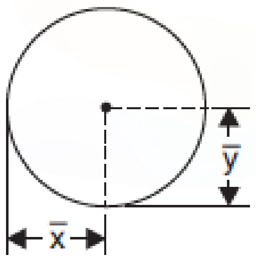
$$x_c = \frac{t\rho(\Delta A_1 x_1+\Delta A_2 x_2+\Delta A_3 x_3+ \ldots\Delta A_n x_n)}{t\rho(\Delta A_1+\Delta A_2+\Delta A_3+ \ldots\Delta A_n)} = \frac{\sum \Delta A_i(x_i)}{\sum(\Delta A)_i}$$



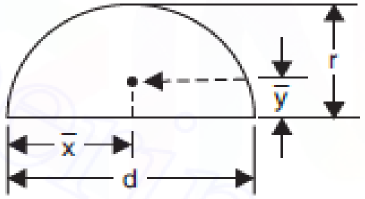
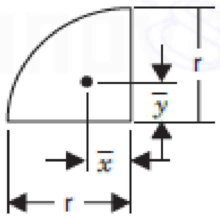
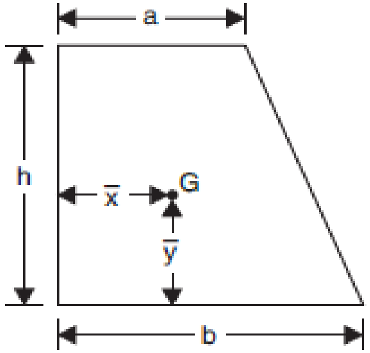
CENTROID TWO DIMENSIONAL BODY

- Similarly, $\boldsymbol{y_c} = \frac{\sum \boldsymbol{\Delta A_i (y_i)}}{\sum (\boldsymbol{\Delta A})_i}$
- x_c and y_c are the coordinates of the centroid of the plate generally called as the coordinates of the centroid of an area.
- Again, as the density and thickness of the plate are constant over the entire area, the coordinates of the centre of gravity become the coordinates of the centroid of the area.
- Generally the term centroid is used for the centre of gravity of a geometrical figure and the term centre of gravity is used when referring to actual physical bodies.

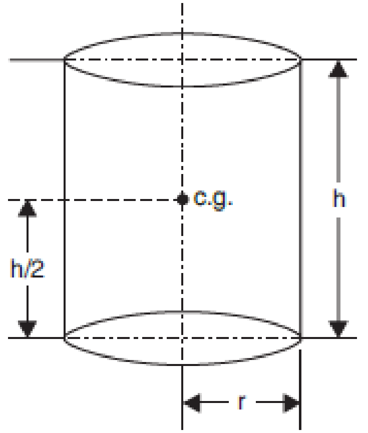
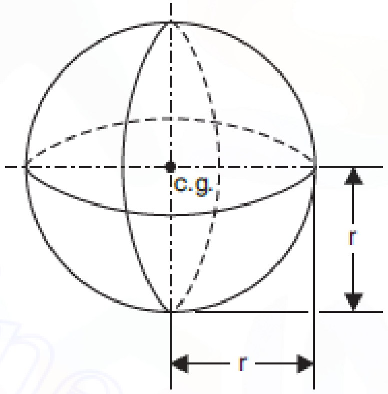
POSITIONS OF CENTROIDS OF PLANE GEOMETRICAL FIGURES

Shape	*Area*	$\bar{x}$	$\bar{y}$	*Figures*
Rectangle	bh	$\frac{b}{2}$	$\frac{h}{2}$	
Triangle	$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$	
Circle	$\frac{\pi}{4}d^2$	$\frac{d}{2}$	$\frac{d}{2}$	

POSITIONS OF CENTROIDS OF PLANE GEOMETRICAL FIGURES

Shape	*Area*	$\bar{x}$	$\bar{y}$	*Figures*
Semicircle	$\frac{\pi}{8}d^2$	$\frac{d}{2}$	$\frac{4r}{3\pi}\ (= 0.424r)$	
Quadrant	$\frac{\pi}{16}d^2$	$0.424\ r$	$0.424\ r$	
Trapezium	$(a+b)\,\frac{h}{2}$	$\frac{a^2+b^2+ab}{3(a+b)}$	$\frac{(2a+b)}{(a+b)} \times \frac{h}{3}$	

POSITIONS OF CENTRE OF GRAVITY OF REGULAR SOLIDS

Shape	*Volume*	*Regular solids*
Cylinder	$\pi r^2 h$	
Sphere	$\frac{4}{3}\pi r^3$	

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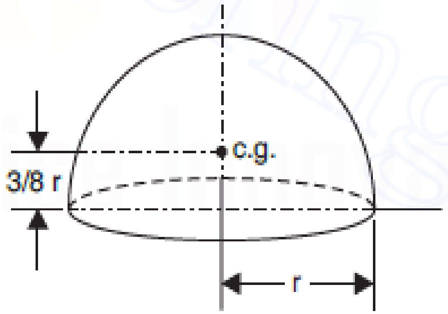
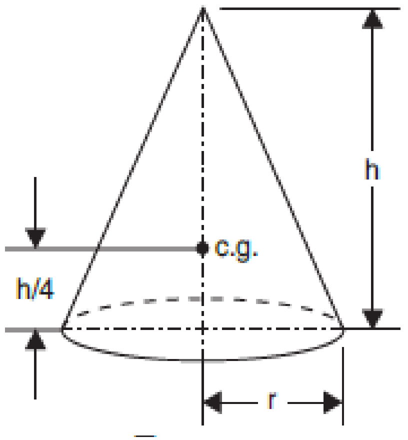
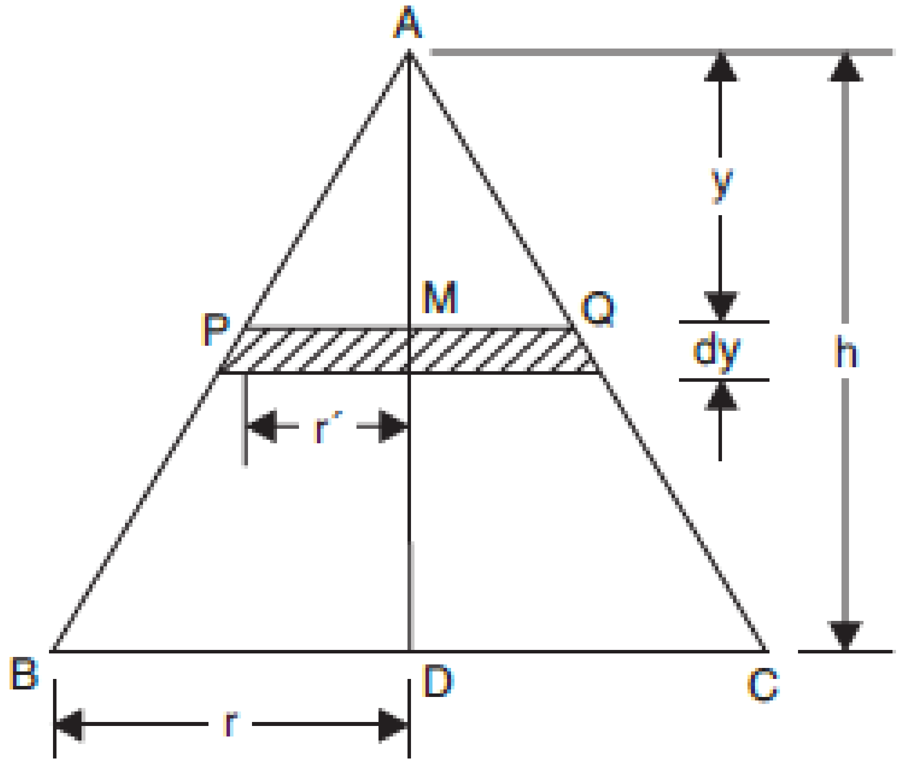
POSITIONS OF CENTRE OF GRAVITY OF REGULAR SOLIDS

Shape	*Volume*	*Regular solids*
Hemisphere	$\frac{2}{3}\pi r^3$	
Right circular cone	$\frac{1}{3}\pi r^2 h$	

CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID RIGHT CIRCULAR CONE

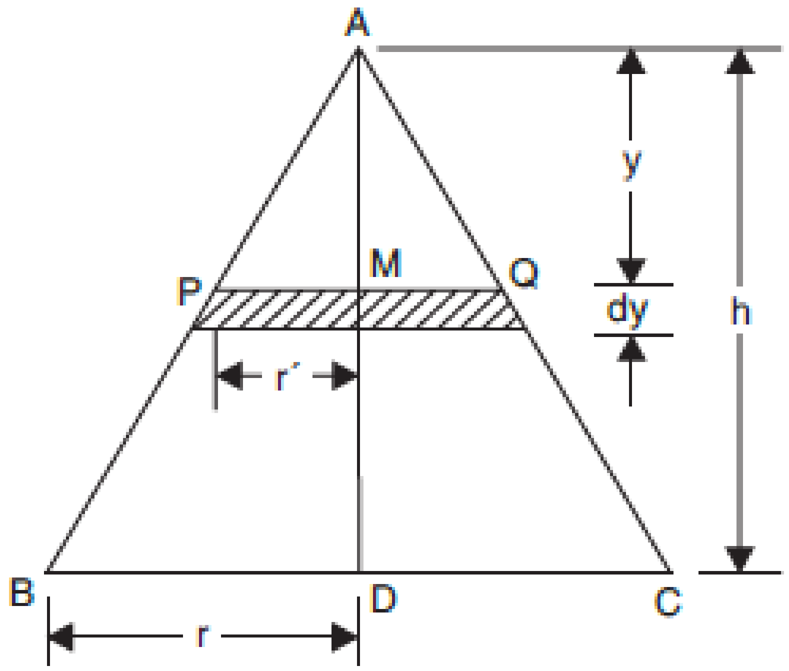


CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID RIGHT CIRCULAR CONE

- Let *ABC* be the cone and *AD* its axis.
- Consider an elementary circular plate *PQ* cut off by two planes parallel to the base *BC* at distance *y* and *y* + *dy* from *A*, and having its centre at *M*.
- Let *AD* = *h*, *BD* = *r*, *PM* = *r*′
- Triangles *APM*, *ABD* are similar.

$$\therefore \frac{AM}{MP} = \frac{AD}{BD}$$

$$\therefore \frac{y}{r'} = \frac{h}{r} \; i.e., r' = \frac{yr}{h}$$



- If *w* be the density of the material, mass of *PQ*

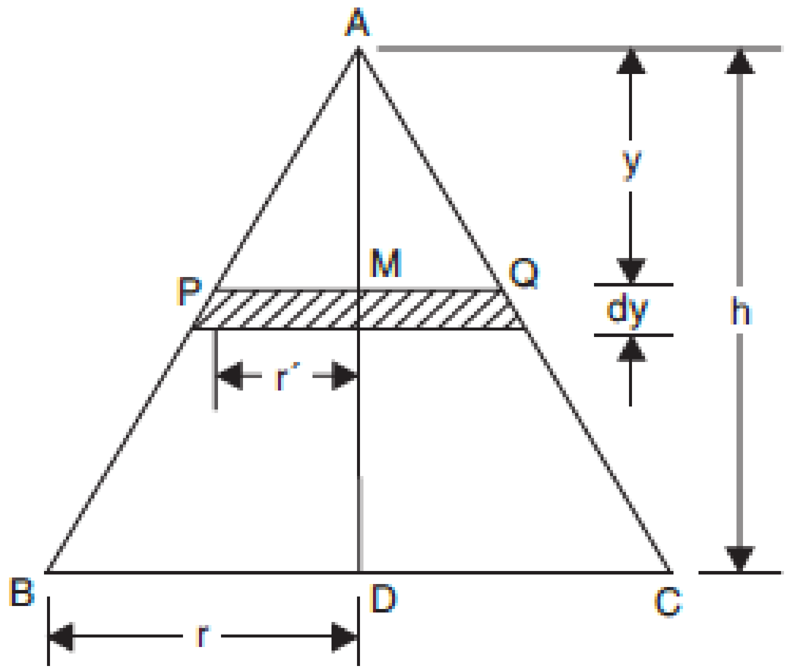
$$= \pi r'2\, dy\,.w$$

$$= \frac{\pi r^2 y^2}{h^2}\, dy\,.w$$

CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID RIGHT CIRCULAR CONE

- The c.g. of *PQ* is at *M*.
- Hence, the distance of c.g. of the cone from *A*

$$= \frac{\sum_{y=0}^{y=h} \frac{\pi r^2 y^2}{h^2} dy.w.y}{\sum_{y=0}^{y=h} \frac{\pi r^2 y^2}{h^2} dy.w} = \frac{\int_0^h y^3 dy}{\int_0^h y^2 dy}$$



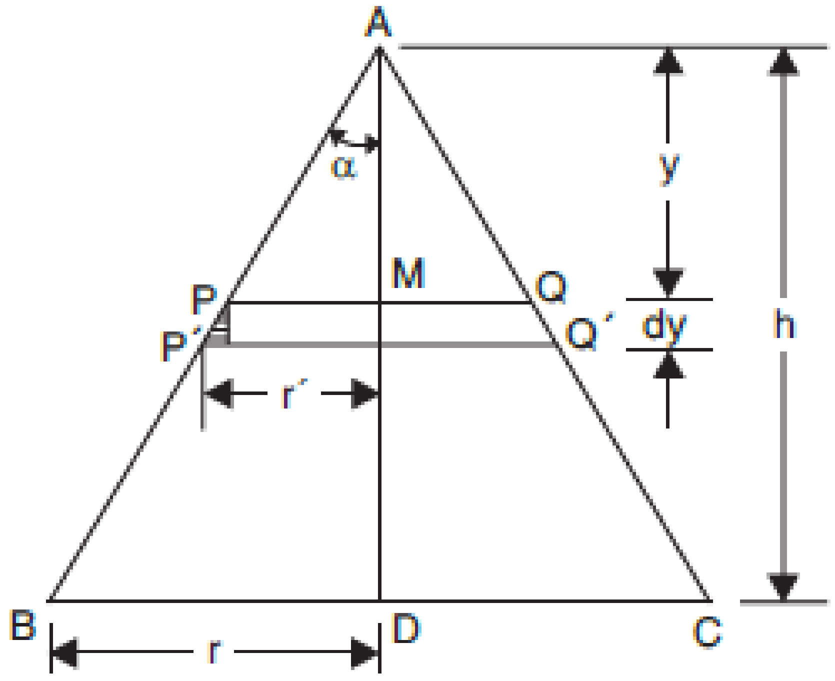
- where, w= weight density = ρg (where, ρ is mass density)

$$= \frac{\frac{h^4}{4}}{\frac{h^3}{3}} = \frac{3h}{4}$$

$$= \left(h - \frac{3h}{4}\right) = \frac{h}{4} \; from\ the\ base$$

- *Hence, the c.g. of a solid cone lies on the axis at a height one-fourth of the total height from the base.*

CENTRE OF GRAVITY IN A FEW SIMPLE CASES: THIN HOLLOW RIGHT CIRCULAR CONE



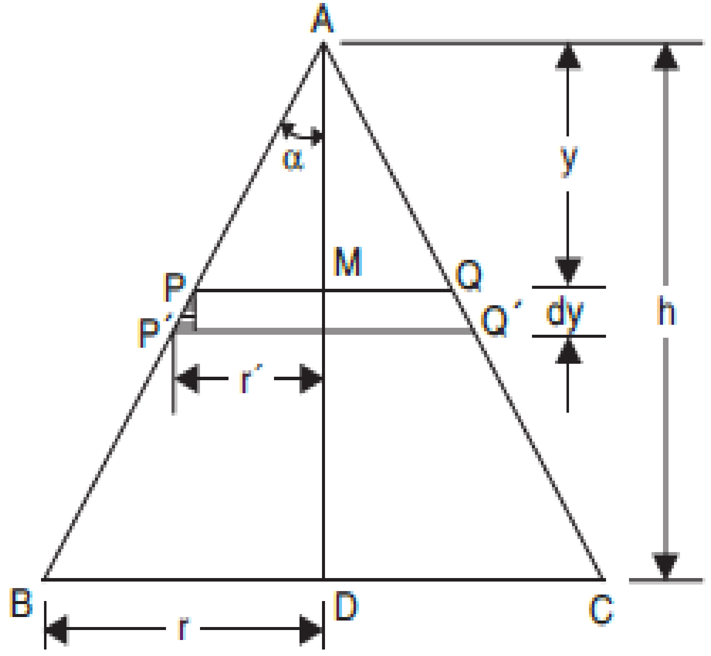
CENTRE OF GRAVITY IN A FEW SIMPLE CASES: THIN HOLLOW RIGHT CIRCULAR CONE

- Let *ABC* be the cone and *AD* its axis.
- Consider a circular ring cut off by planes *PQ* and *P′Q′* to parallel to the base *BC* at distances *y* and *y* + *dy* from *A*.
- Let the radius of *PQ* = *r′ BD* = *r*, *AD* = *h*
- Semi-vertical angle of the cone = ∠*BAD* = α
- Clearly, $PP' = dy \sec\alpha$
- Also $r' = \frac{yr}{h}$ ...(Refer previous case)
- Area of elementary ring = $2\pi r' PP'$

$$= 2\pi \frac{yr}{h} dy \sec\alpha$$

- If *w* be the weight per unit area of the material, weight of the ring

$$= 2\pi \frac{yr}{h} dy \sec\alpha \, . w$$



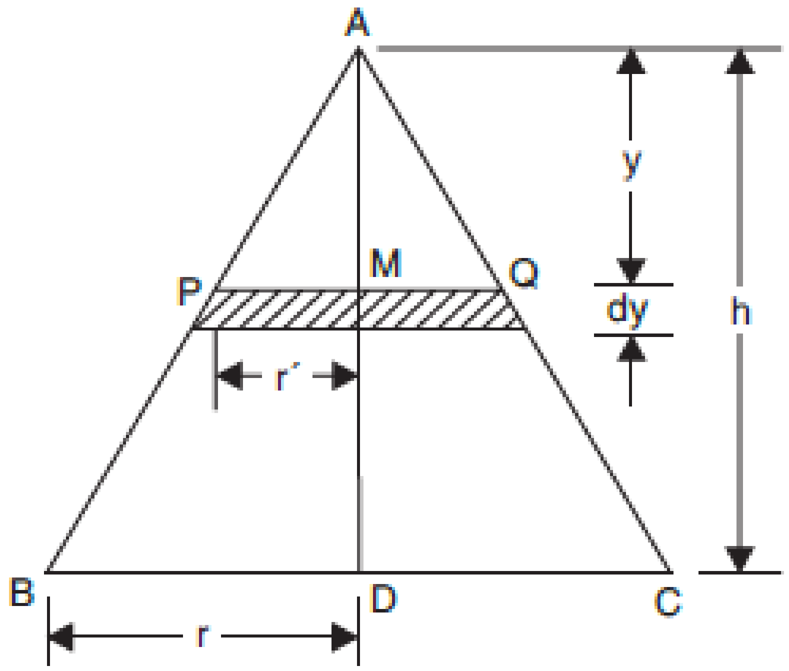
CENTRE OF GRAVITY IN A FEW SIMPLE CASES: THIN HOLLOW RIGHT CIRCULAR CONE

- The c.g. of the ring lies on *AD* at distance *y* from *A*.
- Hence, the distance of the c.g. of the cone from *A*

$$= \frac{\sum_{y=0}^{y=h} 2\pi \frac{yr}{h} dy \sec\alpha\, w . y}{\sum_{y=0}^{y=h} 2\pi \frac{yr}{h} dy \sec\alpha\, w} = \frac{\int_0^h y^2 dy}{\int_0^h y\, dy}$$

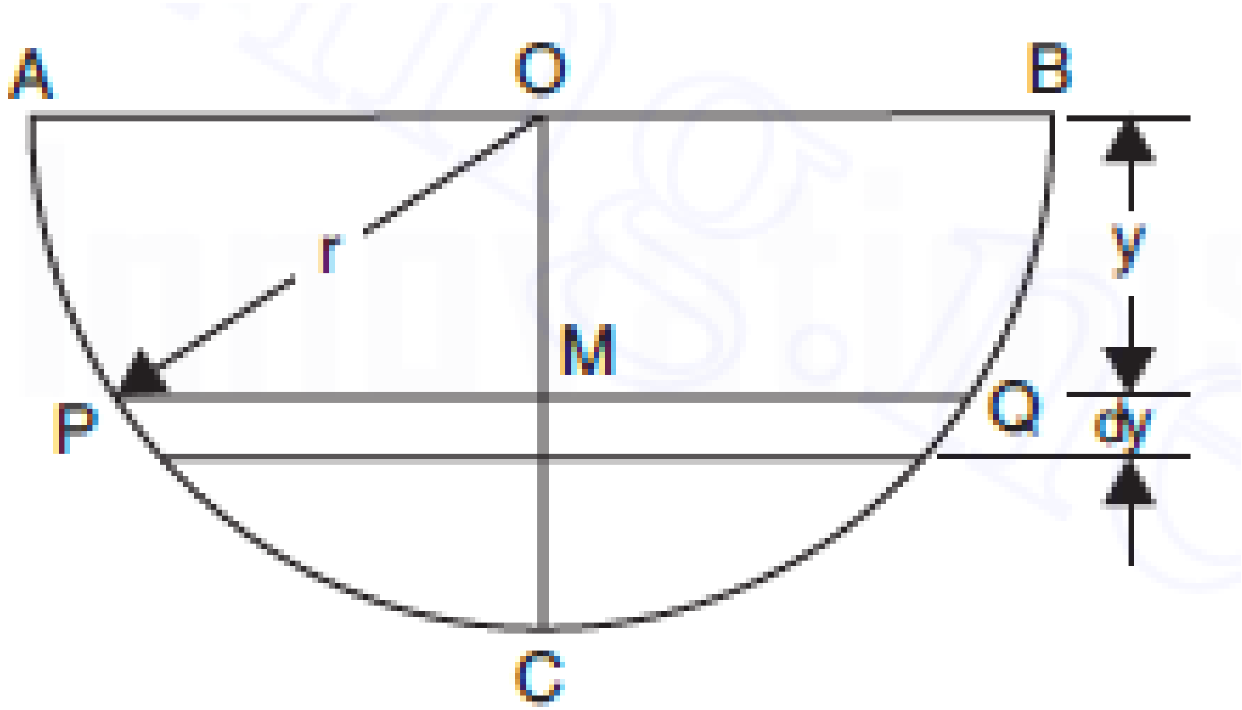
$$= \frac{\frac{h^3}{3}}{\frac{h^2}{2}} = \frac{2h}{3}$$

$$= \left(h - \frac{2h}{3}\right) = \frac{h}{3}\ from\ the\ base$$



- *Hence the c.g. of a thin hollow cone lies on the axis at a height one-third of the total height above the base.*

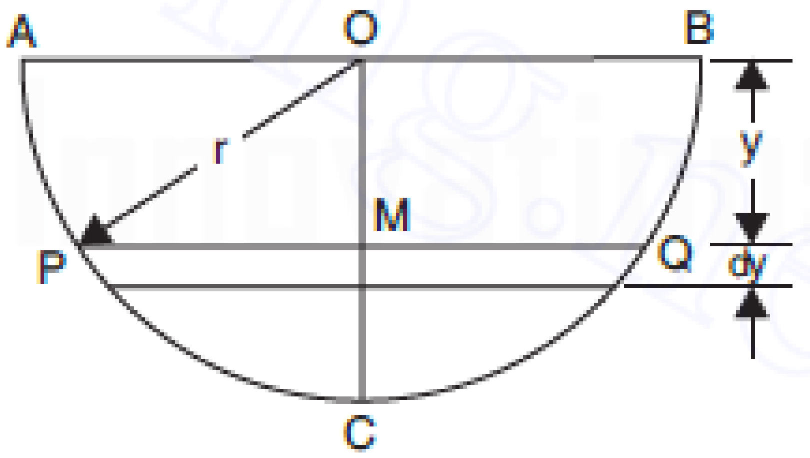
CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID HEMISPHERE



CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID HEMISPHERE

- Let *ACB* be the hemisphere of radius *r*, and *OC* its central radius.
- Consider an elementary circular plate *PQ* cut off by planes parallel to *AB* at distances *y* and *y* + *dy* from *AB*.

$$PM^2 = OP^2 - OM^2$$
$$= r^2 - y^2$$
$$area = \pi(r^2 - y^2) * dy.\omega$$



- Where, ω is the weight of unit volume of the material
- The c.g. of PQ lies on OC at distance y from O.
- Therefore, the distance of the c.g. of the hemisphere from O

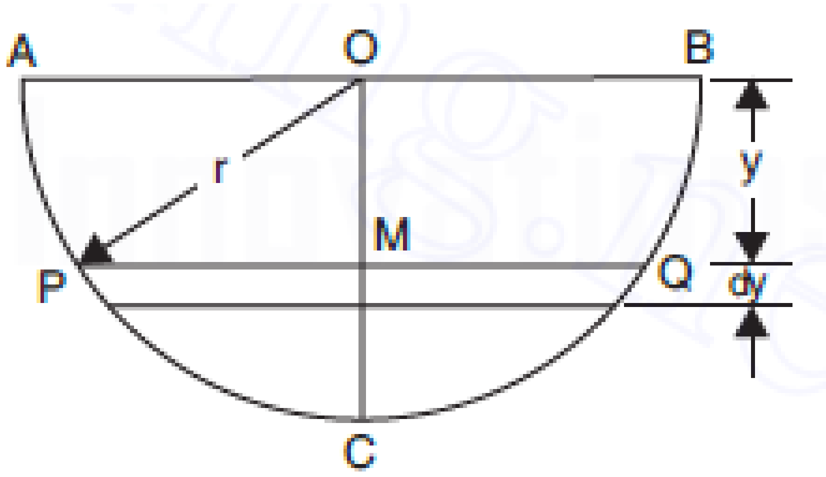
$$= \frac{\sum_{y=0}^{y=r} \pi(r^2 - y^2) * dy.\omega.y}{\sum_{y=0}^{y=r} \pi(r^2 - y^2) * dy.\omega}$$

CENTRE OF GRAVITY IN A FEW SIMPLE CASES: SOLID HEMISPHERE

- The distance of the c.g. of the hemisphere from O

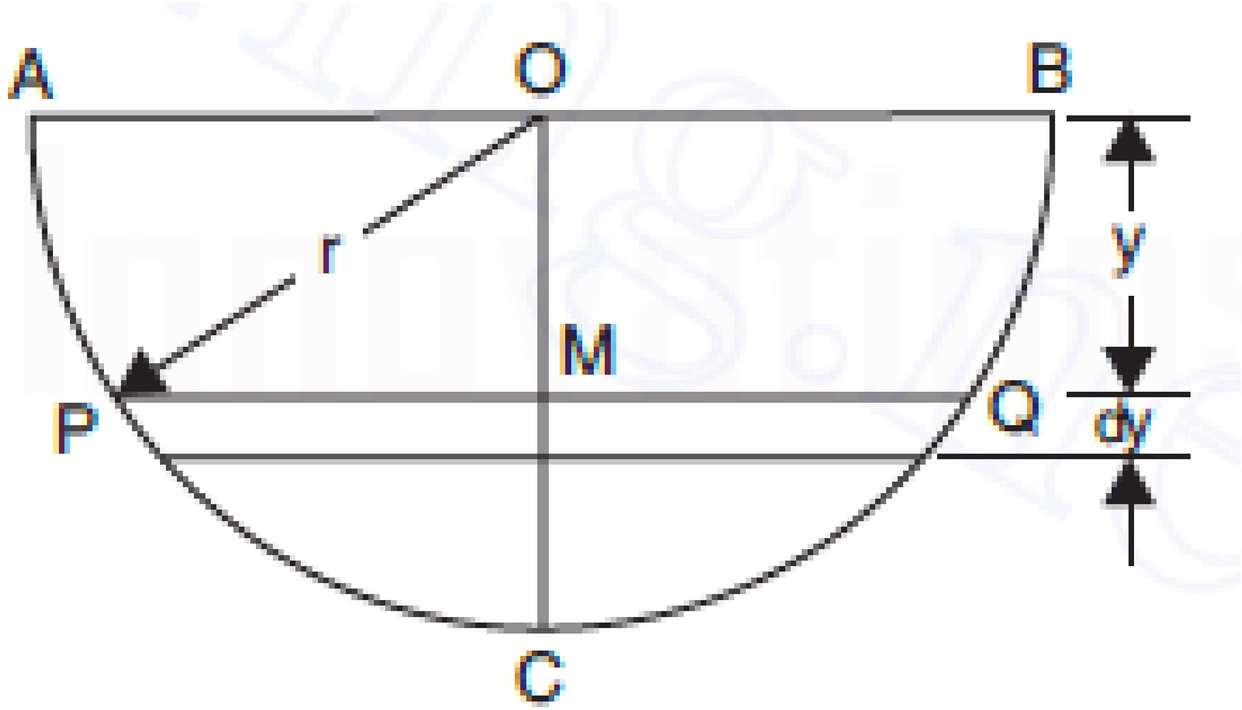
$$= \frac{\int_0^r (r^2 - y^2) y\,dy}{\int_0^r (r^2 - y^2)\,dy} = \frac{\left| \frac{r^2 y^2}{2} - \frac{y^4}{4} \right|_0^r}{\left| r^2 y - \frac{y^3}{3} \right|_0^r}$$

$$= \frac{\frac{r^4}{4}}{\frac{2r^3}{3}} = \frac{3r}{8}$$



- *Hence c.g. of a solid hemisphere lies on the central radius at distance 3r/8 from the plane base,where r is the radius of the hemisphere.*

CENTRE OF GRAVITY IN A FEW SIMPLE CASES: THIN HOLLOW HEMISPHERE



CENTRE OF GRAVITY IN A FEW SIMPLE CASES: THIN HOLLOW HEMISPHERE

- If the hemisphere is hollow of negligible thickness, then *PQ* is a ring whose

$$area = 2\pi r\, dy,$$

by mensuration.

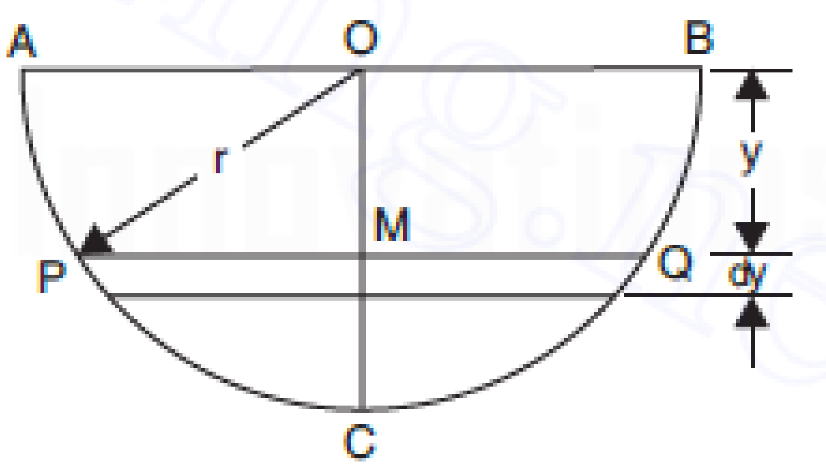
$$Weight\ of\ PQ = 2\pi r\, dy.w$$

- where *w* is the weight per unit area of the material.
- Distance of the c.g. of the hemisphere from $O = \bar{y}$ (say)

$$= \frac{\int_0^r 2\pi r\, dy.w.y}{\int_0^r 2\pi r\, dy.w} = \frac{\int_0^r y.dy}{\int_0^r dy}$$

$$= \frac{\frac{r^2}{2}}{r} = \frac{r}{2}$$

- *Hence the c.g. of a hollow hemisphere bisects the central radius.*



DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD

- By considering a plane figure to be made up of a number of small elements of length or area we cannot generate the true shape of the figure.
- For that we have to make the size of these elements infinitesimally small and their number very large.
- For example, circumference of a circle can be generated only by a polygon having a large number of infinitesimally small sides.
- Mathematically, when the terms **ΔL or ΔA** occurring in the expressions of centre of gravity and centroid become infinitesimally small, the above derived expressions can be written as:

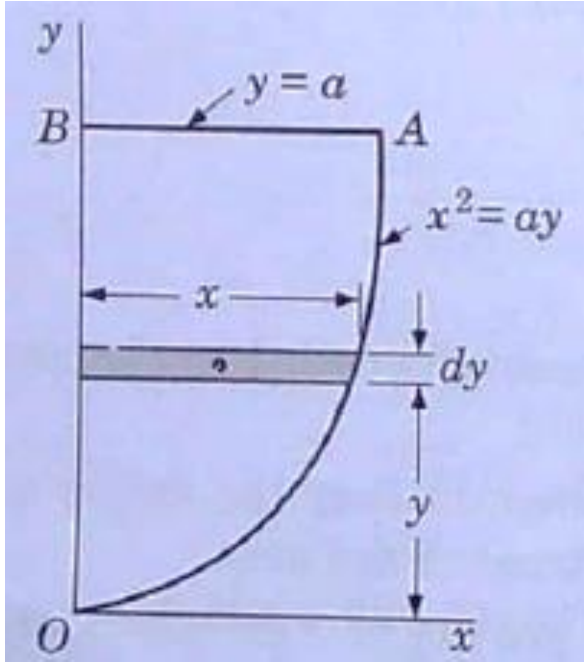
$$x_c = \frac{\int x dL}{\int dL}, y_c = \frac{\int y dL}{\int dL} \quad ; \quad x_c = \frac{\int x dA}{\int dA}, y_c = \frac{\int y dA}{\int dA} \quad ; \quad x_c = \frac{\int x dm}{\int dm}, \; y_c = \frac{\int y dm}{\int dm}$$

- Where, dL, dA and dm denote length, area and mass respectively of a differential element chosen and (x, y) the coordinates of its centroid.
- The integral $\int \boldsymbol{xdA}$ is known as the first moment of area w.r.t. the y-axis.
- While, the integral $\int \boldsymbol{ydA}$ denotes the first moment of area w.r.t. the x-axis.

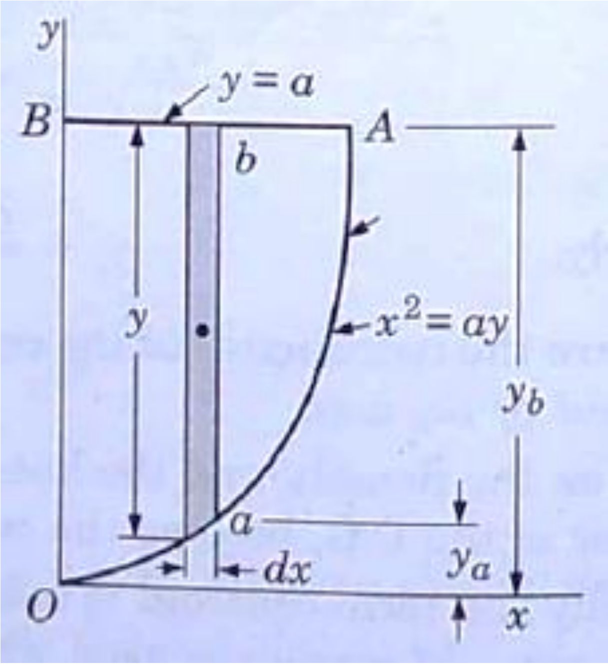
DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD

- Above expressions can be integrated to determine the coordinates of centroid and centre of gravity.
- **Integration method**

A. *Choice of differential element.* In the determination of the coordinates of the centroid by integration method, a proper choice of the differential element can considerably ease and simplify the setting up and the evaluation of the integrals of the type.



$$x_c = \frac{\int x dA}{\int dA}, y_c = \frac{\int y dA}{\int dA}$$



DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD

- Consider an area OAB bounded by the curve **$x^2 = ay$** and straight line **$y = a$.**
- We can consider either a horizontal differential element (strip) or a vertical differential element (strip).
- **For a Horizontal strip**

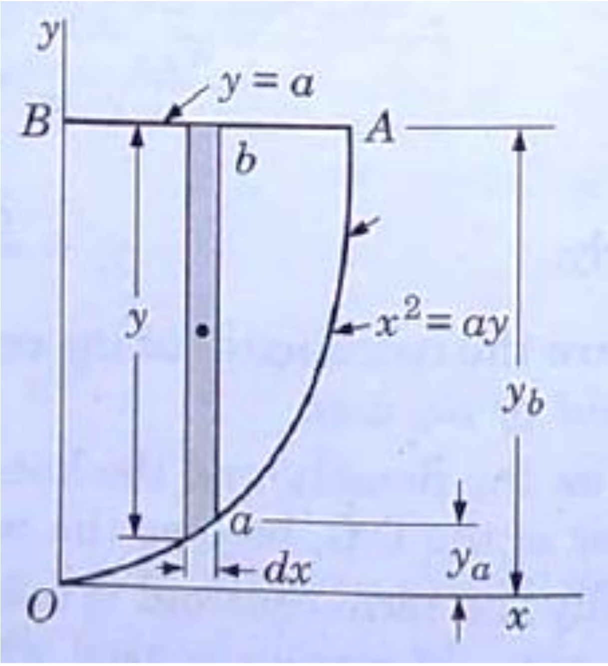
  Area of the differential element, **$dA = xdy$**

  The position of its centroid is $\left(\frac{x}{2}, y\right)$

- **For a vertical strip**

  Area of the differential element, **$dA = ydx$** or **$dA = (y_b - y_a)dx$**

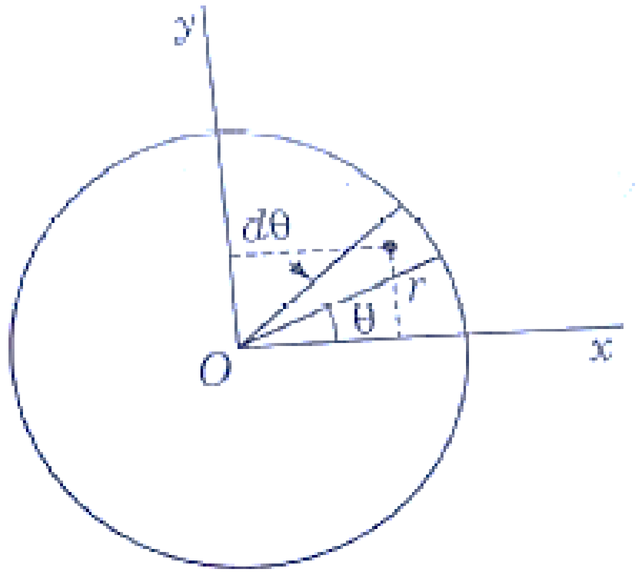
  The position of its centroid is $\left(x, \frac{y_a + yb}{2}\right)$



- It may be noted that the point a lies on the curve OA and the point b on the straight line AB.
- It can be appreciated that of the two elements considered, the horizontal differential element is a better choice for evaluating the integral in the present situation.

DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD

- **Triangular element**
- A triangular element will also be chosen in certain situations as s



Area of the element, $dA = \frac{(rd\theta)r}{2}$

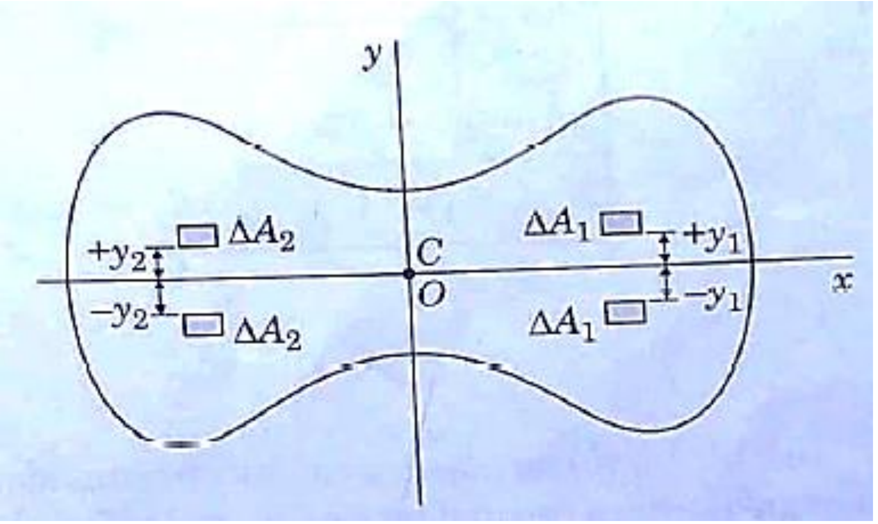
The position of its centroid is = $\left(\frac{2}{3}rcos\theta, \frac{2}{3}rsin\theta\right)$

DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD

B. *Choice of the axis of reference.*

- The determination of the position of the centroid or centre of gravity involves taking the moments of lengths, areas or masses w.r.t. some axis.
- So some frame of reference or coordinate axis are to be chosen.
- Although the location of the centroid or centre of gravity does not depend upon the reference axis chosen, but their proper choice can considerably simplify the calculations.
- Axis of symmetry of a figure, (provided it exists) if chosen as a reference axis can simplify calculations.
- Because if a figure or a curve has some axis of symmetry then the centroid shall lie on this axis of symmetry.

DETERMINATION OF CENTROID AND CENTRE OF GRAVITY BY INTEGRATION METHOD



- Consider a figure having the shape of a dumbbell as shown.
- The figure is symmetrical about the x as well as the y axis.
- Let us discuss the symmetry about the x axis.
- Consider any element of area ΔA, at a distance y_1 above the x-axis.
- Because of the symmetry, there will be a similar element of area ΔA_1 situated at a distance y_1 below the x-axis.
- If sum of moments of all the elements of the area above the x-axis are taken then, it will cancel with sum of moments of similar elements of the area lying below the x-axis.

  Thus, $y_c = 0$ or the centroid lies on the x-axis.
- Similar argument is valid regarding the symmetry about the y-axis.

  Thus, $x_c = 0$, or the centroid lies on the y-axis.

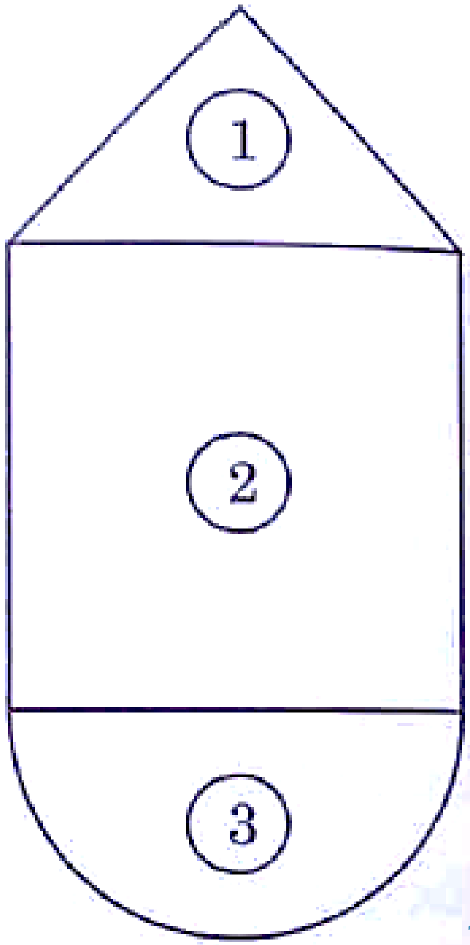
CENTROID COMPOSITE PLANE FIGURE

- A composite area or a curve is one which can be considered to be made up of several pieces or components that represent familiar geometric shapes for example rectangle, circle, triangle, semicircle, eclipse, etc., and for which the positions of individual centroids are known.
- The entire area shown in figure can be considered to be made up of a triangle, rectangle and a semicircle.
- To find centroid of this area, divide area into different component parts.
- Then, we can use the equation,

$$x_c = \frac{\sum \Delta A_i(x_i)}{\sum(\Delta A)_i}, y_c = \frac{\sum \Delta A_i(y_i)}{\sum(\Delta A)_i}$$

Treating each component part as an element of area ΔA_i and the distance of its centroid as (x_i, y_i).

- Positions of centroid of the component parts can be taken as standard results.
- If there is a hole or a void then it may be considered as negative area.



QUESTIONS: FILL IN THE BLANKS :

(*i*) ...... is the force of attraction between a body and the earth and is proportional to mass of the body.

(*ii*) ...... is the point through which the whole weight of a body may be assumed to act.

(*iii*) ...... is the point where the whole area of the figure is assumed to be concentrated.

(*iv*) Centroid of a triangle lies at .... from the base, where h is the height of triangle.

(*v*) Centroid of a semi-circle lies at a distance of ...... from the base, where r is the radius of semi-circle.

(*vi*) Area of a quadrant is given by ......, where r is the radius.

(*vii*) C.G. of a cylinder lies on the axis at a height ...... of the total height from the base.

(*viii*) C.G. of a solid cone lies on the axis at a height ...... of the total height from the base.

(*ix*) C.G. of a hollow cone lies on the axis at a height ...... of the total height above the base.

(*x*) C.G. of a solid hemisphere lies on the central radius at a distance ...... from the plane base, where r is the radius of hemisphere.

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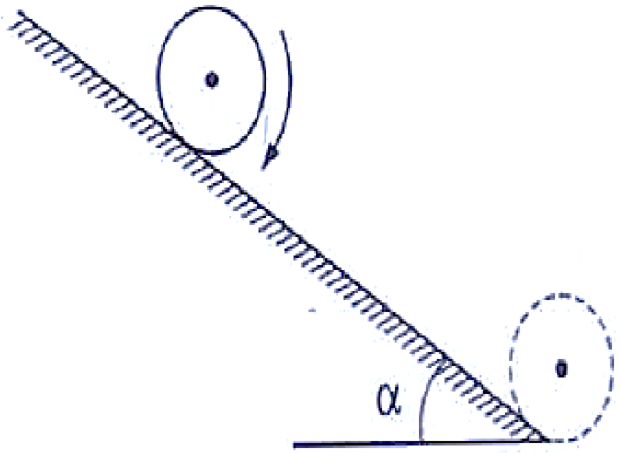
Answers: (*i*) Weight (*ii*) Centre of gravity (*iii*) Centroid (*iv*) $h/3$ (*v*) $0.424r$ (*vi*) $\pi r^2/4$ (*vii*) $h/2$ (*viii*) one-fourth (*ix*) $h/3$ (*x*) $3r/8$.

MOMENT OF INERTIA

- Consider a solid cylinder and a hollow cylinder each of radius r, to slide down (without rolling) as inclined plane of angle α, from rest. Both the bodies shall be observed to reach the bottom of the plane at the same time, experiencing the same acceleration due to gravity (equal to g sin α) irrespective of the mass and radius.



- Concept which gives quantitative estimates of the relative distribution of inertia and mass of a body w.r.t. some reference axis is termed as the moment of inertia of the body.
- i.e., role played by the moment of inertia in the rotary motion is similar to the role played by the mass in the translatory motion.
- The moment of inertia of an area is called the area moment of inertia or the second moment of inertia.
- The moment of inertia of the mass of a body is called the mass moment of inertia.

MOMENT OF INERTIA

- Moment of inertia is defined as the quantity expressed by the body resisting angular acceleration which is the sum of the product of the mass of every particle with its square of a distance from the axis of rotation.
- It can be described as a quantity that decides the amount of torque needed for a specific angular acceleration in a rotational axis. Moment of Inertia is also known as the angular mass or rotational inertia.
- The SI unit of moment of inertia is kg m^2.
- Moment of inertia is usually specified with respect to a chosen axis of rotation. It mainly depends on the distribution of mass around an axis of rotation.
- The moment of inertia depends on the following factors,
    1. The density of the material
    2. Shape and size of the body
    3. Axis of rotation (distribution of mass relative to the axis)

MOMENT OF INERTIA OF AN AREA OF A PLANE FIGURE W.R.T. AN AXIS IN ITS PLANE (RECTANGULAR MOMENTS OF INERTIA)

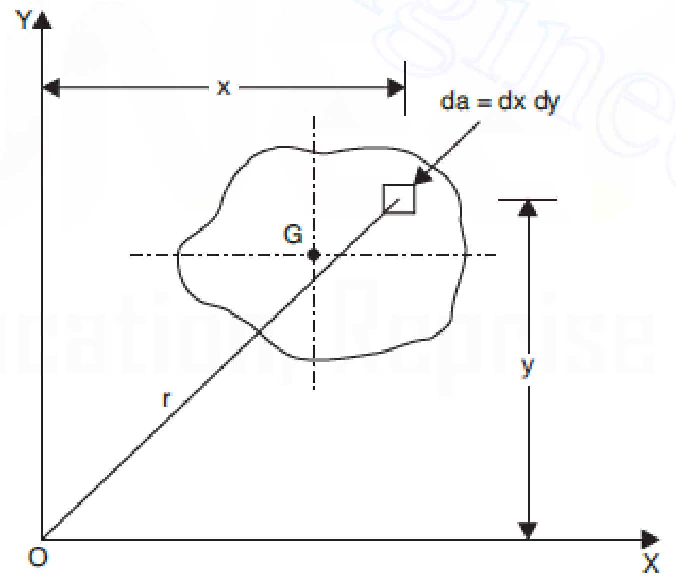
- *The cumulative product of area and square of its distance from an axis is called the moment of inertia of a section about that axis.*
- It can be expressed as

$$I_x = \int_0^A y^2 dA$$

- where I_x = moment of inertia (M.O.I.) of the section about the *x*-axis, and *y* = the distance of infinitesimal area *dA* from the *x*-axis as shown in Figure
- Similarly, the moment of inertia of a section about the *y*-axis is given by

$$I_y = \int_0^A x^2 dA$$

- Integration should cover the entire area of the figure and its value shall depend upon the shape of the area and its orientation w.r.t. the axis.



POLAR MOMENT OF INERTIA

- Moment of inertia of an area of a plane figure w.r.t. an axis perpendicular to the x-y plane and passing through a pole o (z axis) is called the polar moment of inertia and is denoted by I_p.

$$I_p = \int_0^A r^2 dA$$

- As $x^2+y^2 = r^2$, So, $I_p = \int_0^A x^2 dA + y^2 dA = I_x + I_y$
- i.e., *The cumulative product of area and square of its distance from a point is known as the polar moment of inertia.*
- Moment of inertia of an area = (area)*(distance)^2=(Length)^4. Thus, it has the unit of (metre) ^{4}.
- The moment of inertia of an area can be determined w.r.t an axis.
- One commonly used axis is the centroidal axis.
- Any axis passing through the centroid of an area is called the centroidal axis. Two of them are centroidal x axis and centroidal y axis.

RADIUS OF GYRATION

- One of the properties of cross-section which influence the structural behaviour of the members is *radius of gyration*. (Denoted by k_i)

$$k_i = \sqrt{\frac{I_i}{A}}$$

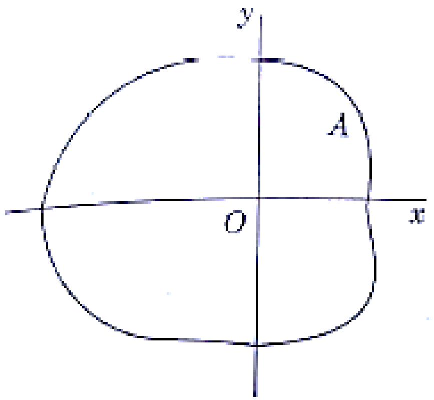
- Where, I_i = moment of inertia about i^{th} axis ; and k_i = radius of gyration of area about i^{th} axis.
- Radius of gyration of a body about an axis of rotation is defined as the radial distance of a point which would have a moment of inertia the same as the body's actual distribution of mass, if the total mass of the body is assumed to be concentrated.
- Members, when subjected to axial forces tend to buckle.
- The load at which members will buckle is proportional to the square of the radius of the gyration.
- The radius of gyration is usually referred to *w.r.t. centroidal axes system of the reaction.*

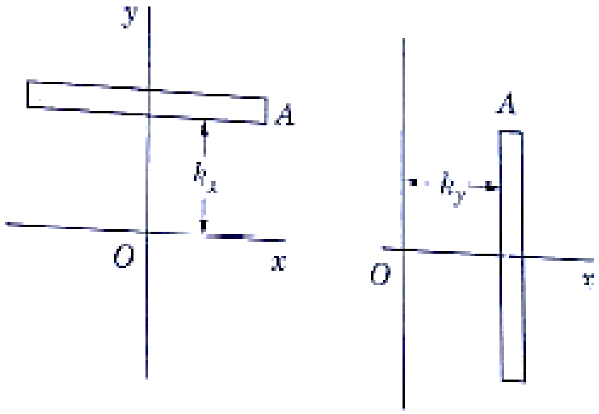
RADIUS OF GYRATION OF AN AREA

- Mathematically the radius of gyration is the root mean square distance of the object's parts from either its center of mass or a given axis, depending on the relevant application.
- It is actually the perpendicular distance from point mass to the axis of rotation. One can represent a trajectory of a moving point as a body. Then radius of gyration can be used to characterize the typical distance travelled by this point.
- Consider an area A which has a moment of inertia I_x w.r.t. the x-axis. Let us imagine this area A to be concentrated into a thin strip parallel to the x-axis.
- If this area A (concentrated strip), is to have the same moment of inertia (I_x) w.r.t. the x-axis, the strip should be placed at a distance k_x from the x axis as given by the relation:

$$I_x = k_x^2 A \text{ or } k_x = \sqrt{\frac{I_x}{A}}$$

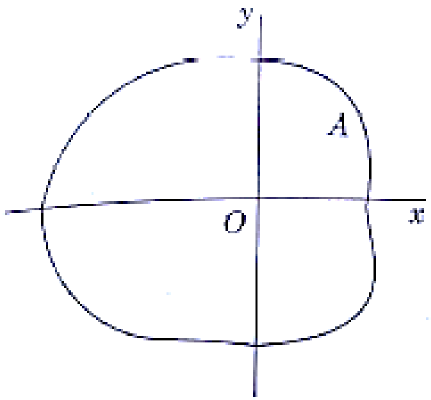
- k_x is known as the radius of gyration of the area w.r.t. the x-axis and has a unit of length (m).





RADIUS OF GYRATION OF AN AREA

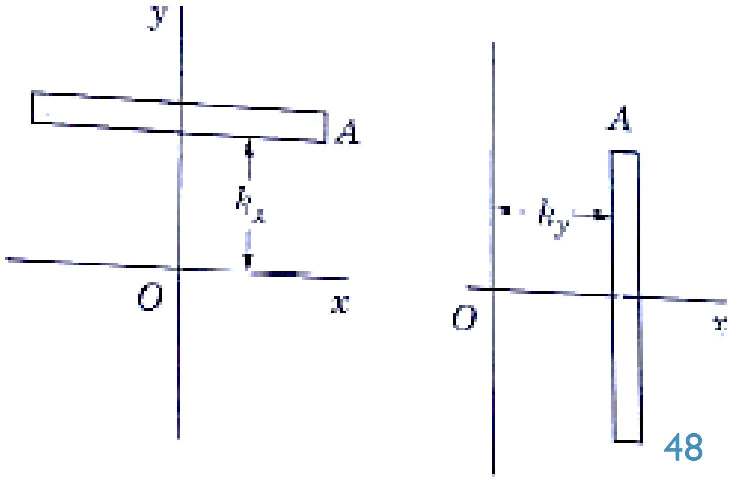
- Radius of gyration w.r.t. the y-axis.



$$I_y = k_y^2 A \text{ or or } k_y = \sqrt{\frac{I_y}{A}}$$

- Where, k_y is known as the radius of gyration of the area w.r.t. the x-axis and has a unit of length (m).
- Radius of gyration w.r.t. the polar axis.

$$k_p = \sqrt{\frac{I_p}{A}}$$



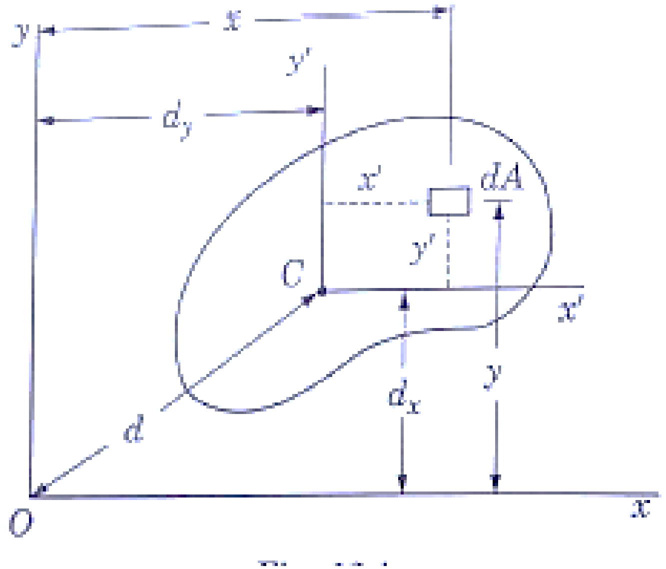
- As, $I_p = I_x + I_y$
- $k_p^2 A = k_x^2 A + k_y^2 A \quad or \qquad k_p^2 = k_x^2 + k_y^2$

PARALLEL AXIS THEOREM

- Let x, y be the rectangular coordinate axis through any point O in the plane of figure of area A as shown in figure.
- x', y' the corresponding parallel axis through the centroid C of the area.
- The axis through the centroid of an area is called the centroidal axis.
- The moment of inertia of the area about the x-axis.

$$I_x = \int_0^A y^2 dA$$

- Where, dA is an element of area at a distance y from the x-axis. But, $y = d_x + y'$



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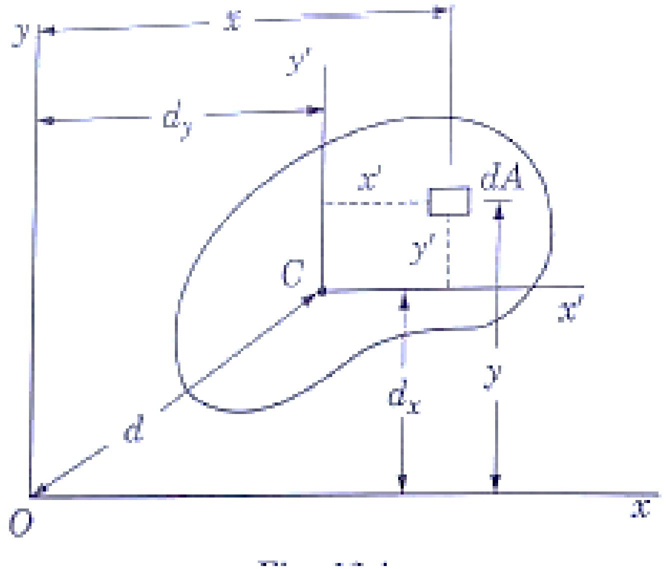
- Where, dA is an element of area at a distance y from the x-axis. But, $y = d_x + y'$
- d_x being the perpendicular distance between the axis x and x'

$$I_x = \int_0^A (y' + dx)^2 dA$$



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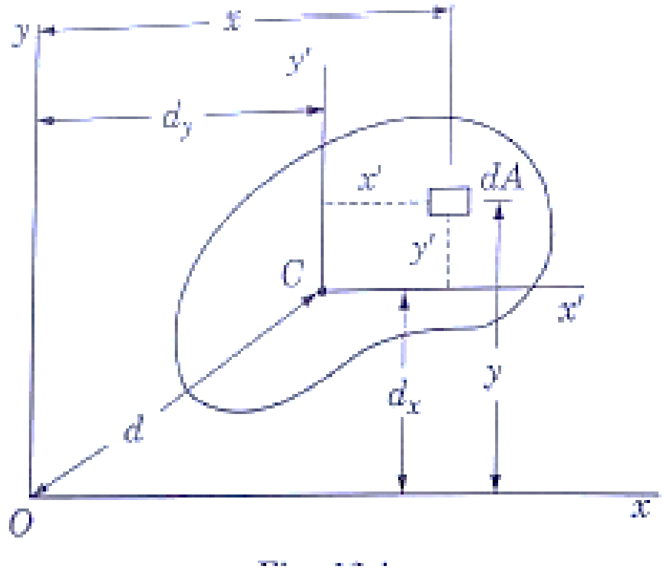

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$$I_x = \int_0^A (y' + dx)^2 dA = \int_0^A (y'^2 + dx^2 + 2y'd_x) dA$$

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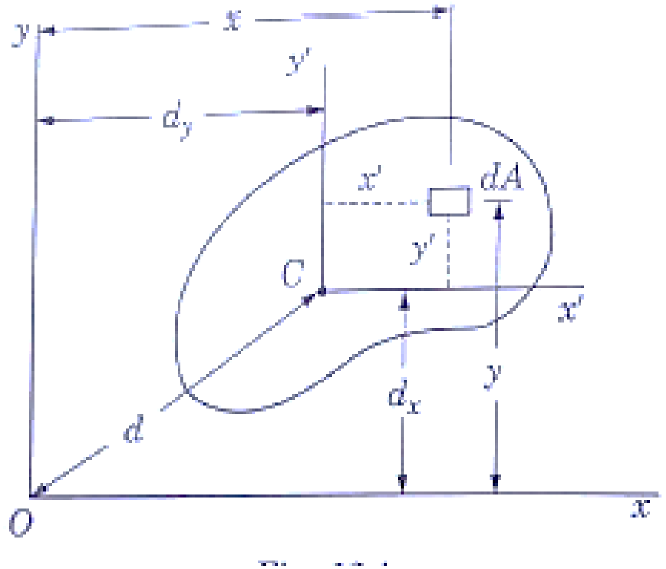

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$$I_x = \int_0^A y'^2 dA + \int_0^A {d_x}^2 dA + \int_0^A 2y'd_x dA =$$

PARALLEL AXIS THEOREM

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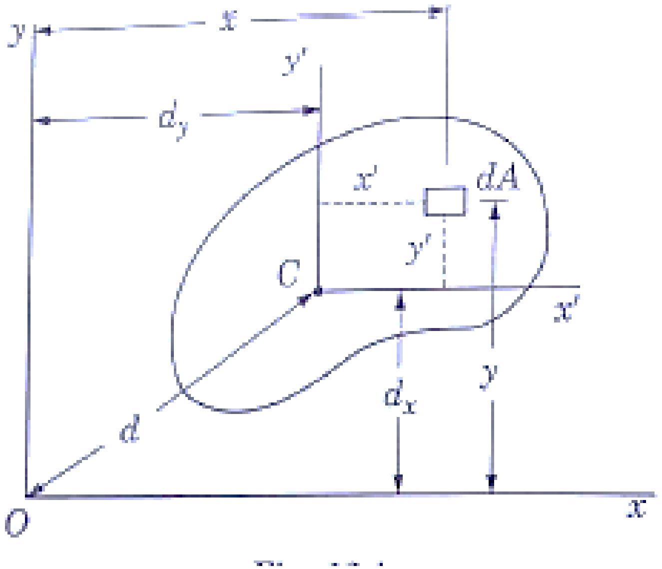

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$$I_x = \int_0^A y'^2 dA + Ad_x{}^2 + 0$$

PARALLEL AXIS THEOREM

- The terms $\int y' dA$ represents the first moment of the area A about its own centroidal axis x', and is therefore 0.
- The term $\int \mathrm{y}'2 \ \mathrm{dA}$ represents the moment of inertia of the area A about the axis x'.

$$I_x = \overline{I_x} + \mathrm{A}(d_x^2) \quad and \quad I_y = \overline{I_y} + \mathrm{A}(d_y^2)$$

- Moment of inertia of an area about its centroidal axis is represented by $\overline{I_x} + \overline{I_y}$.
- Thus, moment of inertia of an area w.r.t. any axis in its plane = moment of inertia of the area w.r.t. a parallel centroidal axis plus product of the area and square of the distance between the two axes.
- Adding I_x and I_y yields:

$$I_x + I_y =$$

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$$I_x + I_y = \overline{I_x} + \overline{I_y} + \mathrm{A}(d_x^2 + d_y^2)$$

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- Thus, moment of inertia of an area w.r.t. any axis in its plane = moment of inertia of the area w.r.t. a parallel centroidal axis plus product of the area and square of the distance between the two axes.
- Adding I_x and I_y yields:

$$I_x + I_y = \overline{I_x} + \overline{I_y} + \mathrm{A}(d_x^2 + d_y^2)$$

$But, (d_x^2 + d_y^2 = d^2)\ \ and\ \overline{I_x} + \overline{I_y} = Ic$

PARALLEL AXIS THEOREM

- The terms $\int y' dA$ represents the first moment of the area A about its own centroidal axis x', and is therefore 0.
- The term $\int \mathrm{y}'2\ \mathrm{dA}$ represents the moment of inertia of the area A about the axis x'.

$$I_x = \overline{I_x} + \mathrm{A}(d_x^2) \quad and \quad I_y = \overline{I_y} + \mathrm{A}(d_y^2)$$

- Moment of inertia of an area about its centroidal axis is represented by $\overline{I_x} + \overline{I_y}$.
- Thus, moment of inertia of an area w.r.t. any axis in its plane = moment of inertia of the area w.r.t. a parallel centroidal axis plus product of the area and square of the distance between the two axes.
- Adding I_x and I_y yields:

$$I_x + I_y = \overline{I_x} + \overline{I_y} + \mathrm{A}(d_x^2 + d_y^2)$$

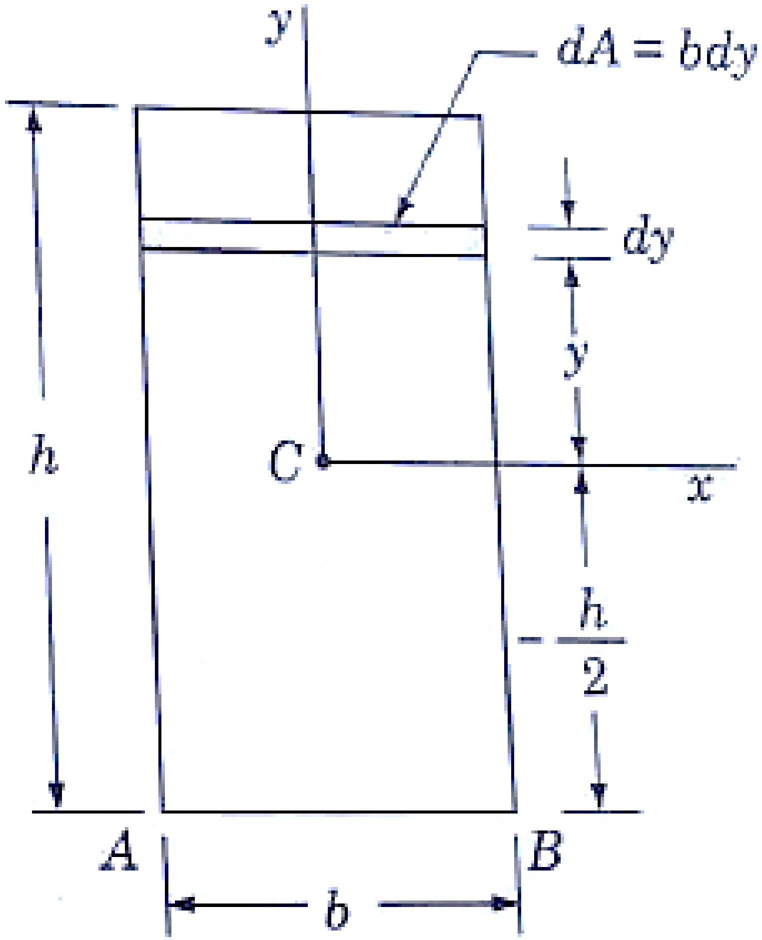
$But,\ (d_x^2 + d_y^2 = d^2)\ \ and\ \overline{I_x} + \overline{I_y} = Ic$

$$I_x + I_y = I_p$$

- Where J_c is the polar moment of inertia about the centroidal axis. Therefore, $I_p = I_c + Ad^2$
- Thus the parallel axis theorem is applicable to the polar moment of inertia also.

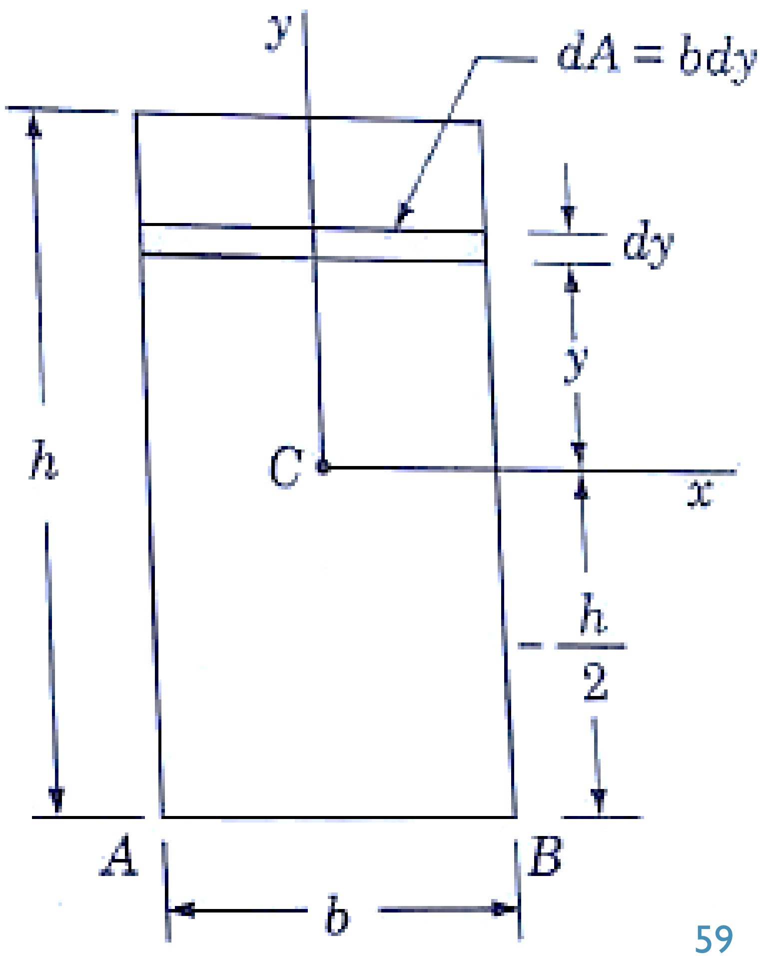
PARALLEL AXIS THEOREM : EXAMPLE

- Example 1. Find the moment of inertia of a rectangular cross section about its centroidal Axis as shown. Also, find its moment of inertia about the base AB.



PARALLEL AXIS THEOREM : EXAMPLE

- The centroid of the rectangular area is at C. Centroidal Axis x-y are as shown and the area is symmetrical about both these axes.



Moment of inertia about the centroidal axis.

- Consider an element of thickness dy situated at a distance y from the x-axis.
- The area of the element, **dA = bdy**
- Moment of inertia of the elemental area about the x-axis is

$$dI_x = y^2(bdy)$$

$$\overline{I_x} = \int dI_x = \int_{y=-h/2}^{y=+h/2} by^2 dy = b\left[\frac{y^3}{3}\right]_{-h/2}^{+h/2}$$

- $\overline{I_x} = \frac{bh^3}{12}$
- $\overline{I_y} = \frac{hb^3}{12}$

PARALLEL AXIS THEOREM : EXAMPLE

Moment of inertia about the base.

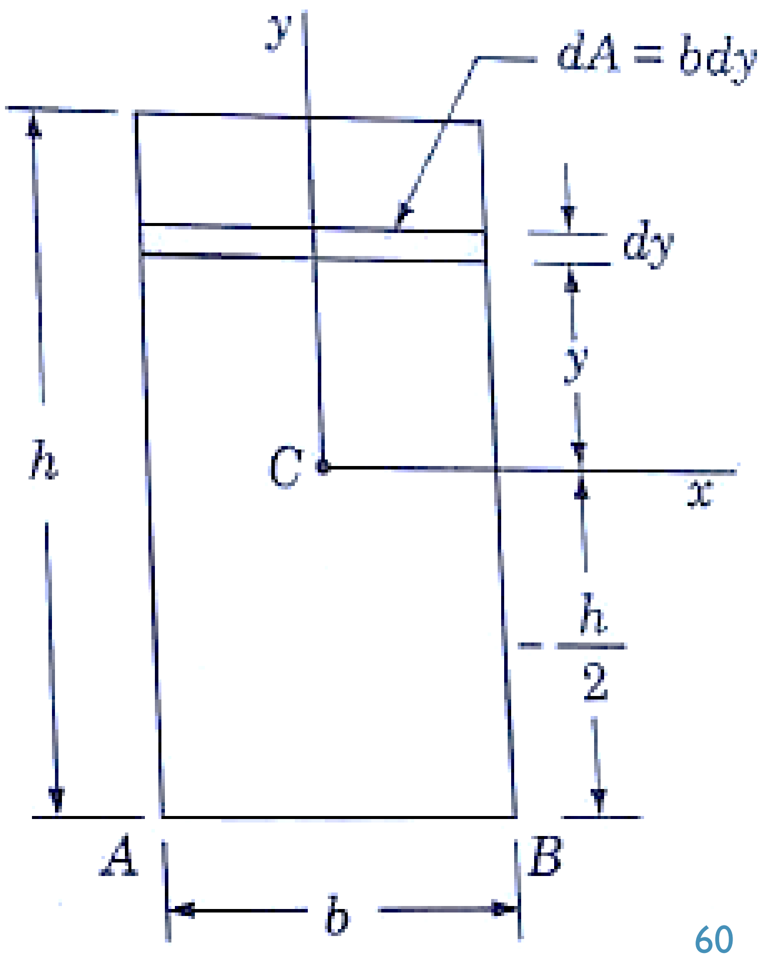
- Using parallel axis theorem,

$$I_{AB} = \overline{I_x} + Ad^2$$

- where, d is the perpendicular distance of the centroid C from the base AB.

$$I_{AB} = \frac{bh^3}{12} + (bh)\left(\frac{h}{2}\right)^2$$

$$I_{AB} = \frac{bh^3}{3}$$



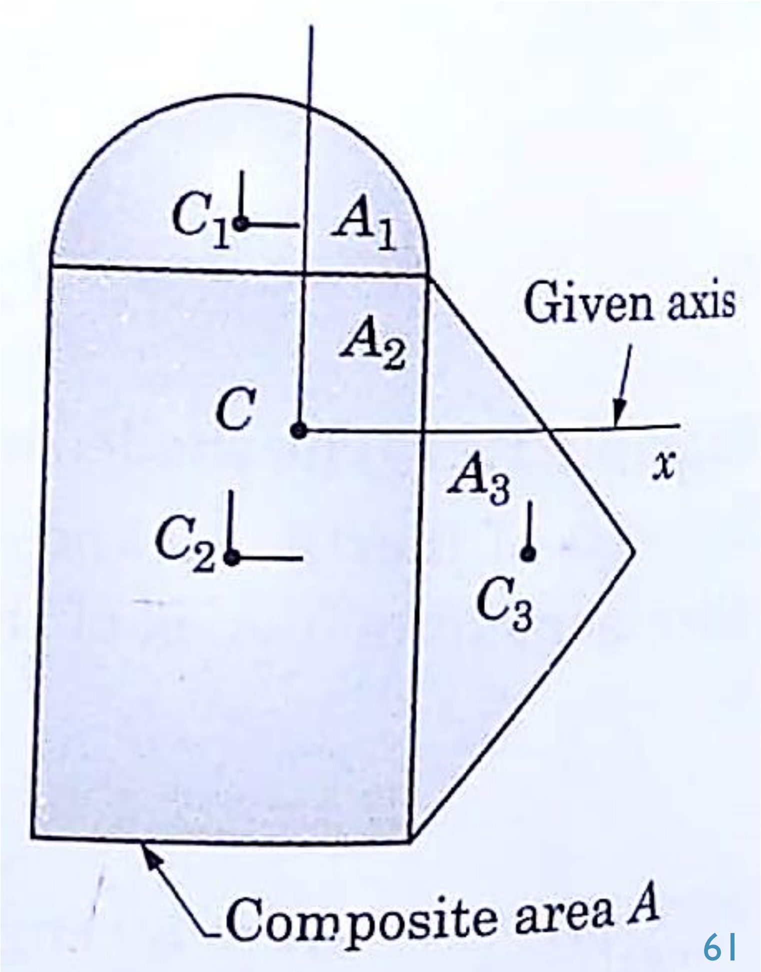
MOMENT OF INERTIA OF A COMPOSITE AREA/HOLLOW SECTION

- A composite area is one which can be considered to be made up of several components of an area of familiar geometric shapes.
- Consider a composite area A as shown in figure.
- It can be considered to be made up of three component areas A_1, A_2 and A_3; a semicircle, a rectangle and triangle respectively.
- The moment of inertia of the composite area about an axis is related to the moment of inertia of the component areas as,

 The moment of inertia of an area w.r.t. a given Axis

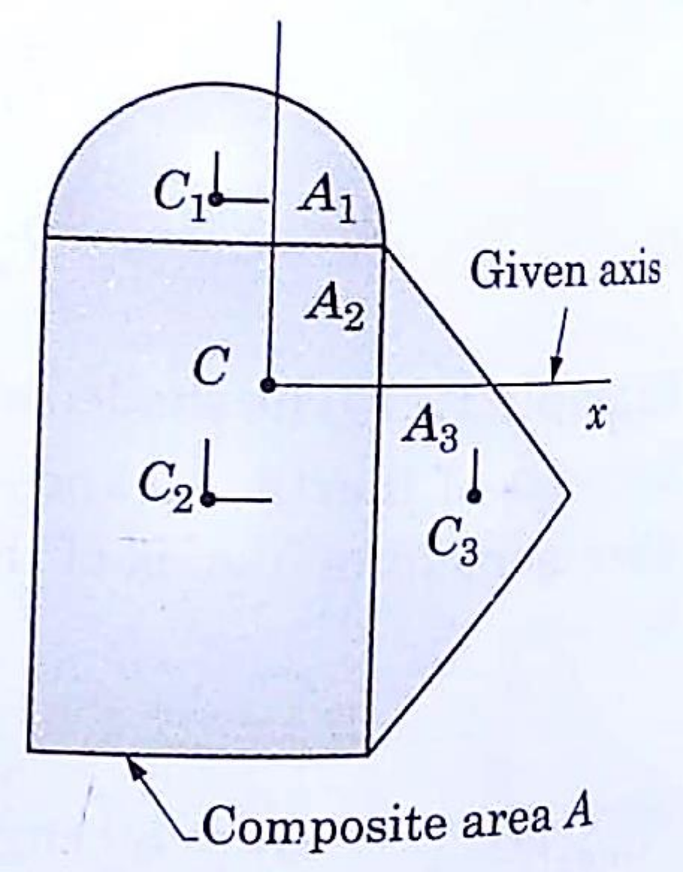
 = the sum of the moment of inertia of the component areas (A_1, A_2 and A_3) w.r.t. the same axis.

- Quite often it is required to determine the moment of inertia of a composite area w.r.t. an axis parallel through the centroid of the composite area called the centroidal axis of the composite area.



MOMENT OF INERTIA OF A COMPOSITE AREA/HOLLOW SECTION: STEPS

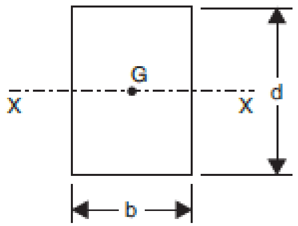
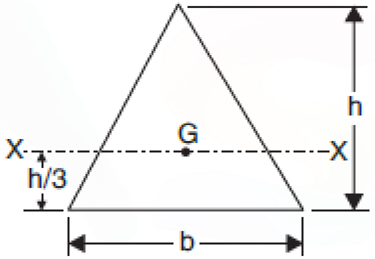
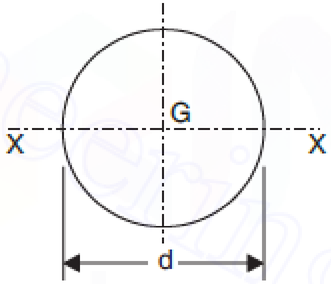
1. Split up the given area A into component areas of familiar shapes. Determine the values of the component areas A_1, A_2, A_3……and locate the positions of their individual centroids C_1, C_2, C_3.
2. Locate the centroid C of the composite area.
3. Calculate the moment of inertia of each component area (A_1) about an axis passing through its centroid (C_1)and parallel to the given axis (x).
4. Transfer these moments of inertia of component areas, to the given axis through the centroid C of the composite area A, using the parallel axis theorem.
5. Add the moments of inertia of component areas to obtain the moment of inertia of the composite area.
6. If a composite area consists of a component area which represents a void, hole or an area removed then its moment of inertia is negative and has to be subtracted.



Radius of gyration of a composite area is not equal to sum of radii of the variation of component areas.

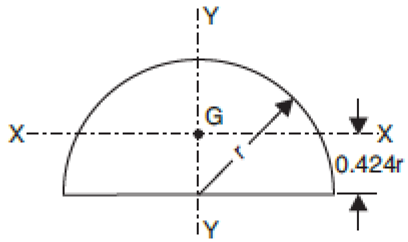
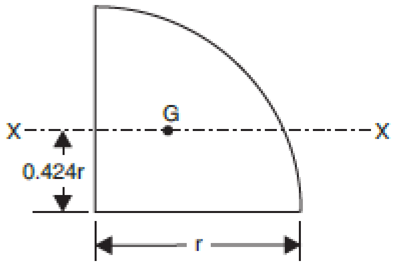
MOMENT OF INERTIA OF SIMPLE AREAS

Table 5.1. Moments of Inertia for Simple Areas

Shape	*Moment of inertia*	*Simple areas*
Rectangle	$I_{xx} = \dfrac{bd^3}{12}$	 Fig. 5.2
Triangle	$I_{xx} = \dfrac{bh^3}{36}$	 Fig. 5.3
Circle	$I_{xx} = \dfrac{\pi d^4}{64}$	 Fig. 5.4

MOMENT OF INERTIA OF SIMPLE AREAS

Table 5.1. Moments of Inertia for Simple Areas

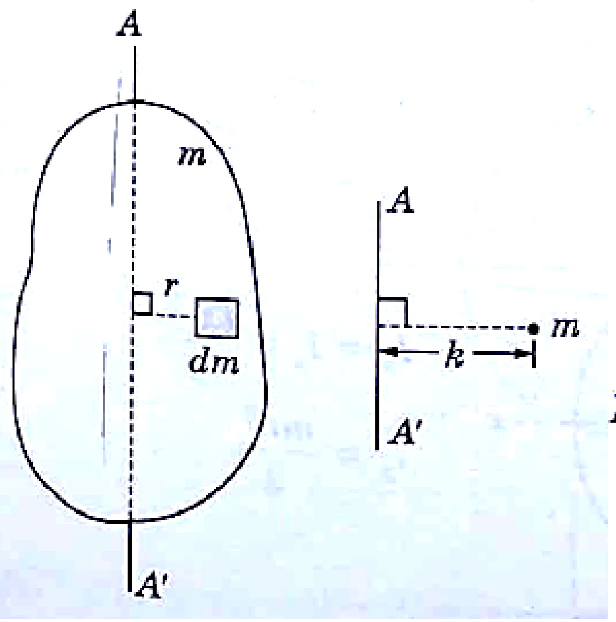
Shape	*Moment of inertia*	*Simple areas*
Semi-circle	$I_{xx} = 0.11r^4$ $I_{yy} = \dfrac{\pi d^4}{128}$	 Fig. 5.5
Quadrant	$I_{xx} = 0.055r^4$	 Fig. 5.6

MOMENT OF INERTIA OF A MASS OF RIGID BODY

- Consider a body of mass m. The moment of inertia of the body w.r.t. the axis AA' is defined by integral

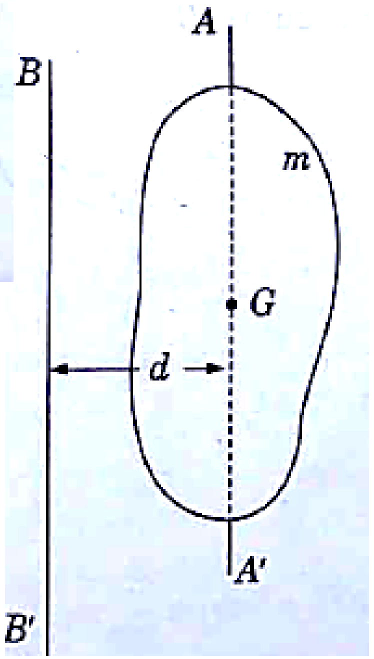
$$I = \int_0^m r^2 dm$$

where, dm is the mass of an element of the body situated at a distance r from the axis AA' and the integration is extended over the entire volume of the body.



- Thus, moment of inertia of a body has the dimension of mass * (length) ^2
- The radius of gyration k of body w.r.t axis AA' is given by the relation

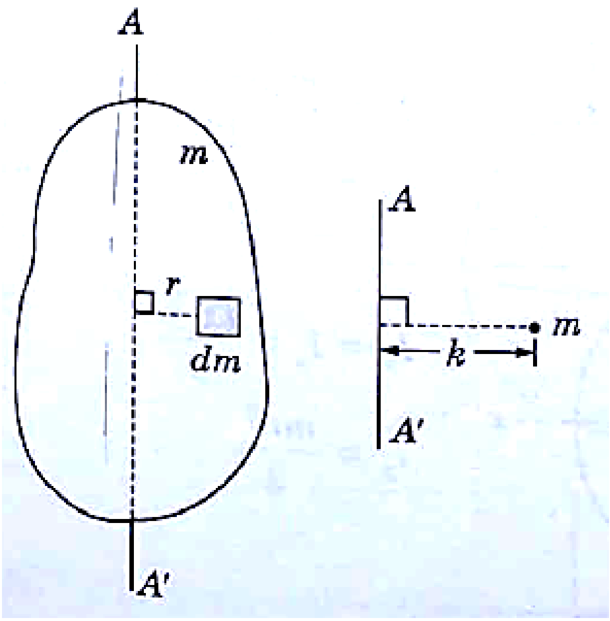
$$I = k^2 m \; or \; k = \sqrt{\frac{I}{m}}$$



MASS MOMENT OF INERTIA-PARALLEL AXIS THEOREM

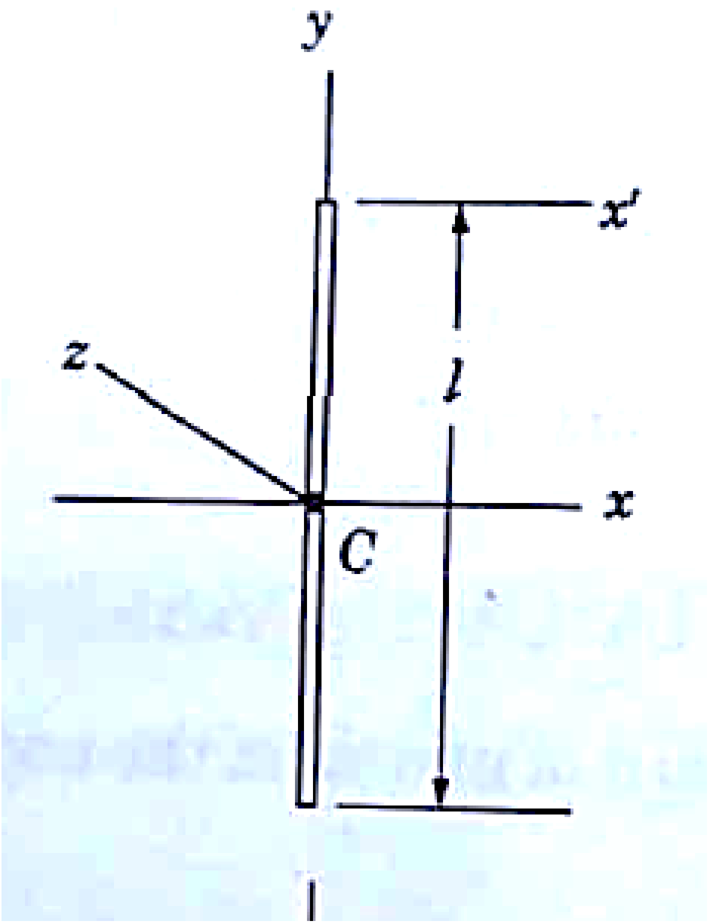
- Consider a body of mass m.
- Let the moment of inertia of the body w.r.t. an axis AA' passing through the centre of gravity G of the body $\bar{I}$.
- Then the moment of the inertia I of the body w.r.t. axis BB' which is parallel to the centroidal axis AA' and is at a distance d from it is given by

$$I = \bar{I} + md^2$$



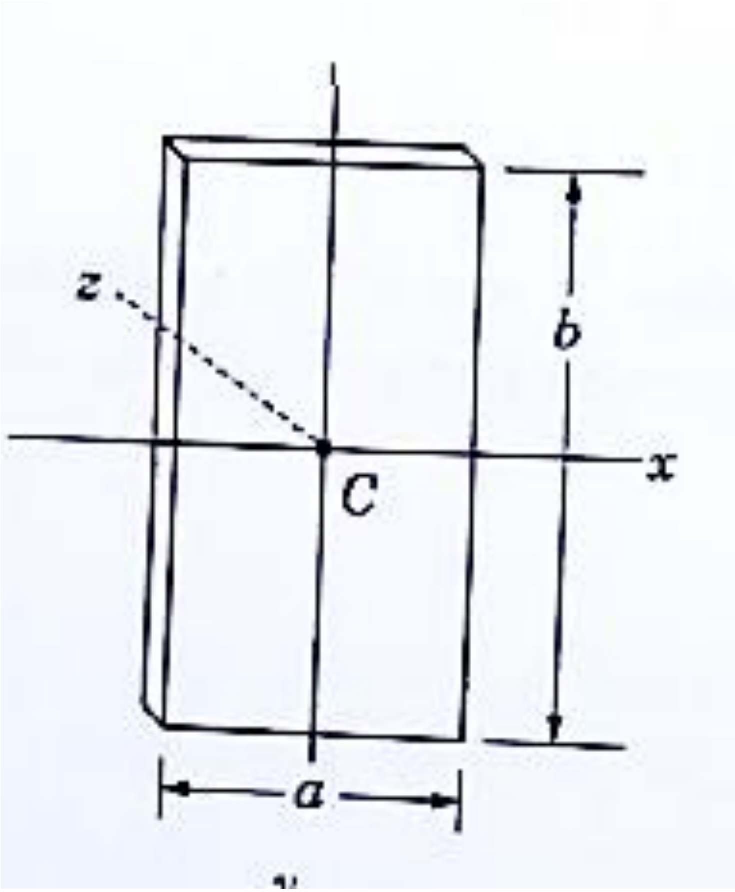
MASS MOMENT OF INERTIA OF SLENDER ROD

- $I_x = \frac{ml^2}{12}$
- $I'_x = \frac{ml^2}{3}$
- $I_y = 0$
- $I_z = I_x = \frac{ml^2}{12}$



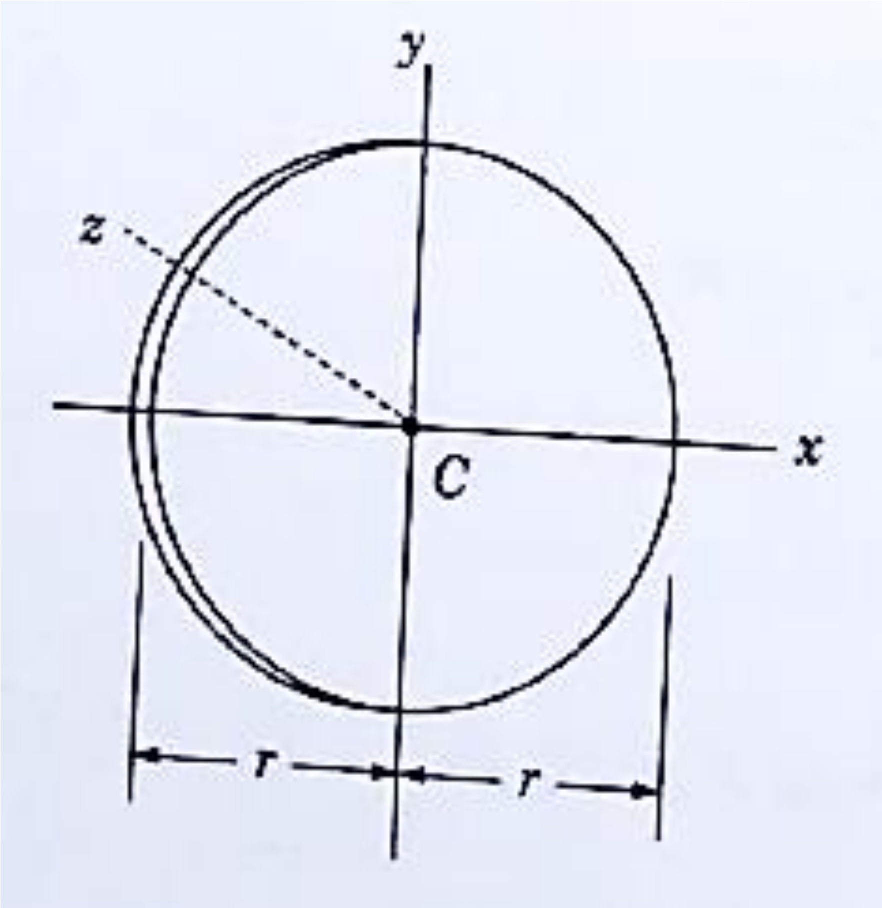
MASS MOMENT OF INERTIA OF THIN RECTANGULAR PLATE

- $I_x = \frac{mb^2}{12}$
- $I_y = \frac{ma^2}{12}$
- $I_z = I_x + I_y$
- $I_z = \frac{mb^2}{12} + \frac{ma^2}{12}$
- $I_z = \frac{m(a^2+b^2)}{12}$



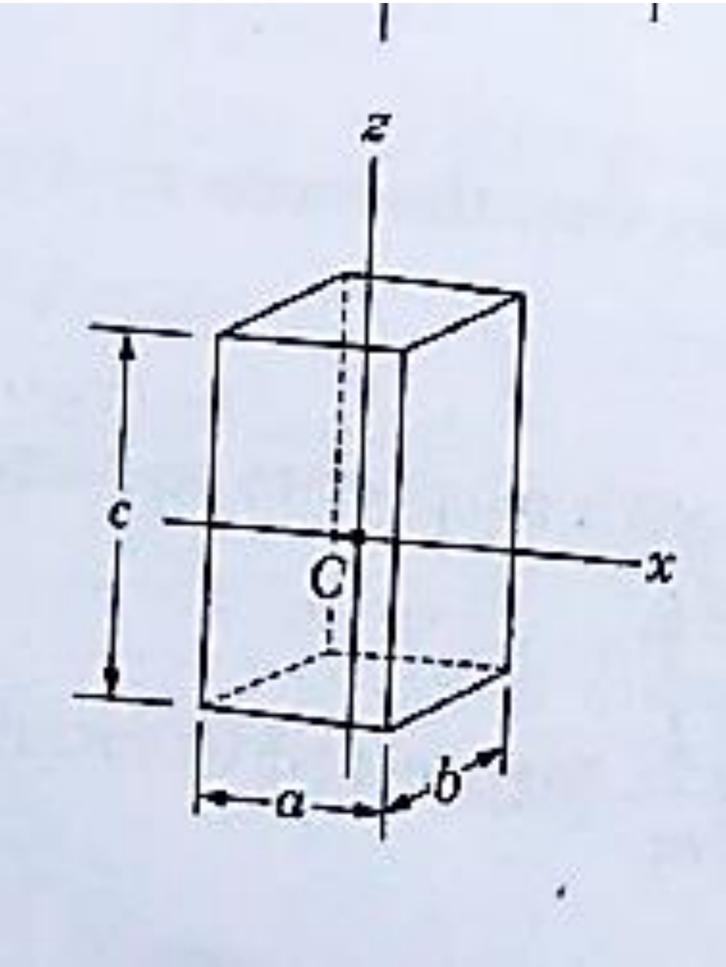
MASS MOMENT OF INERTIA OF THIN DISC

- $I_x = \frac{mr^2}{4}$
- $I_y = \frac{mr^2}{4}$
- $I_z = I_x + I_y$
- $I_z = \frac{mr^2}{4} + \frac{mr^2}{4}$
- $I_z = \frac{mr^2}{2}$



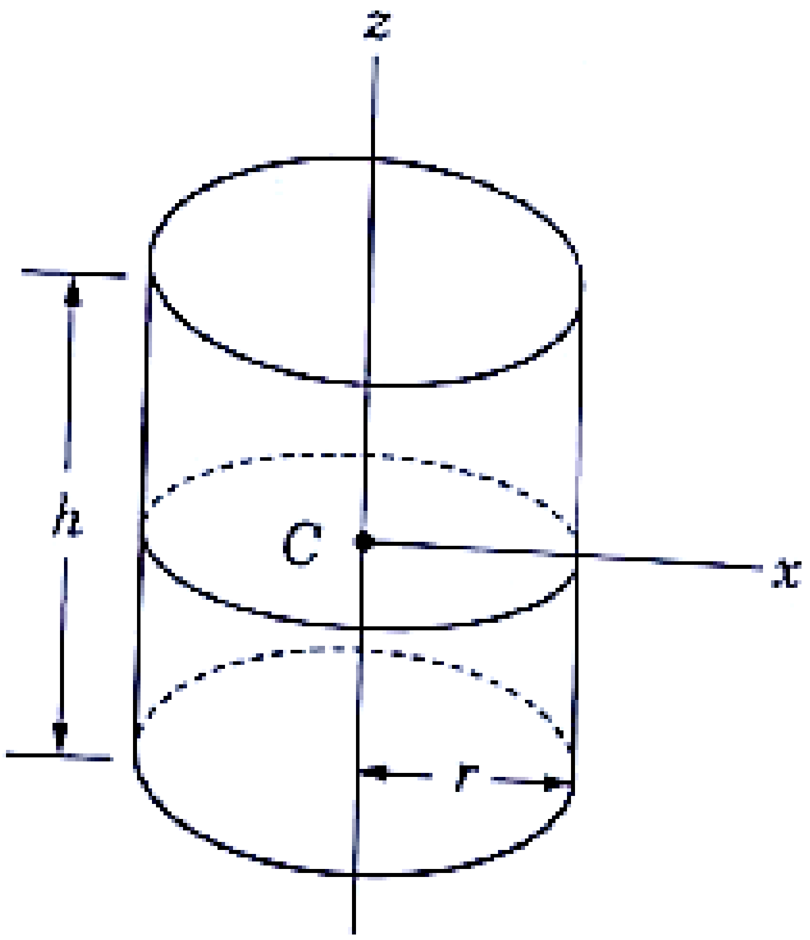
MOMENT OF INERTIA OF A MASS OF RECTANGULAR PRISM

- $I_x = \frac{m(b^2+c^2)}{12}$
- $I_z = \frac{m(a^2+b^2)}{12}$



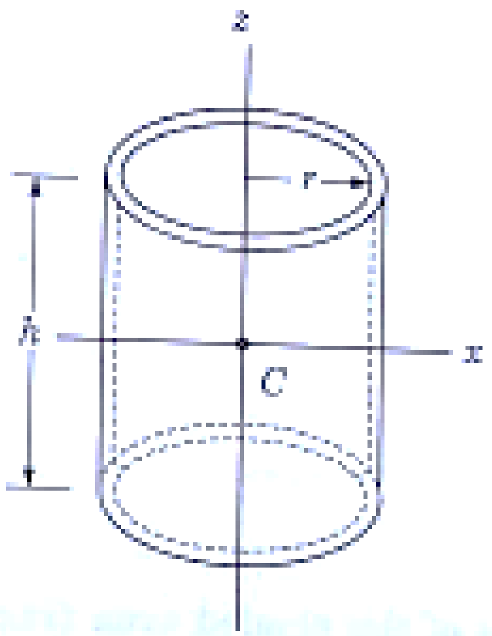
MOMENT OF INERTIA OF A MASS OF CIRCULAR CYLINDER

- $I_x = \frac{m(3r^2 + h^2)}{12}$
- $I_z = \frac{m(r^2)}{2}$



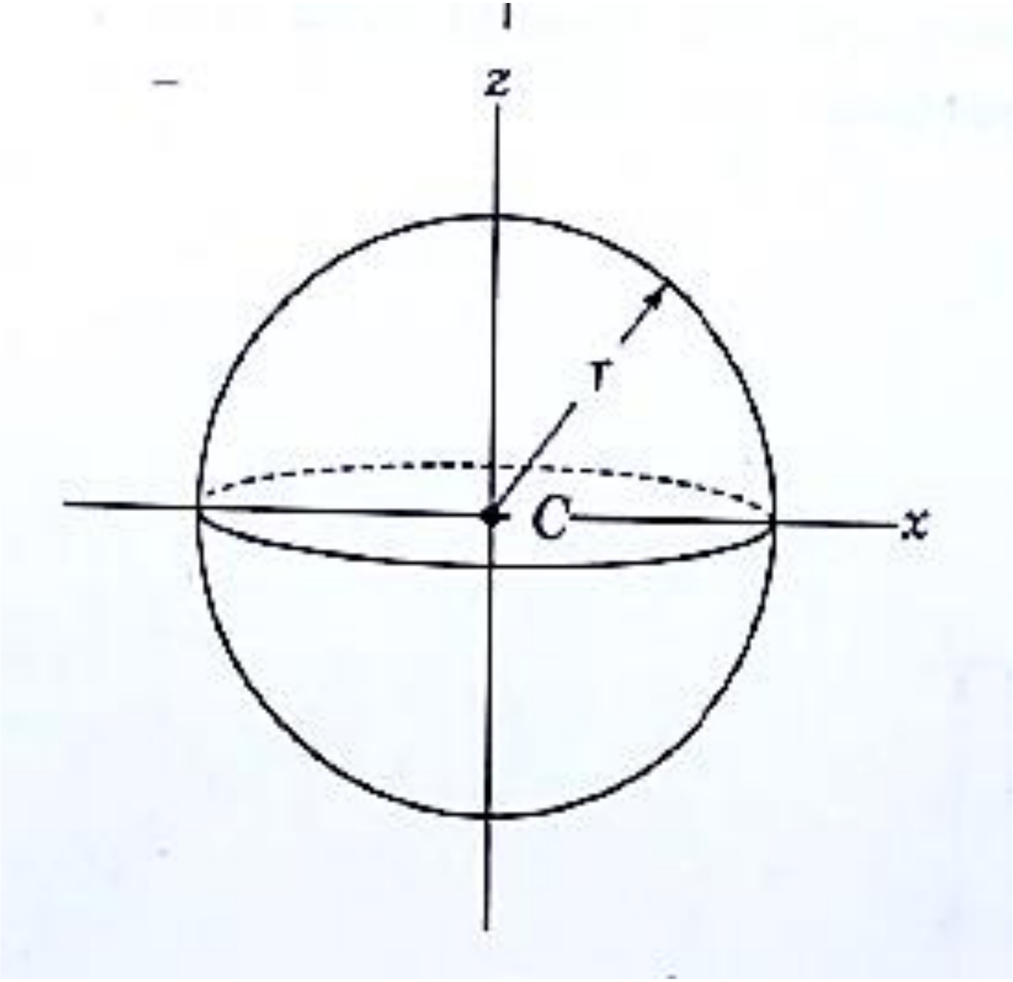
MOMENT OF INERTIA OF A MASS OF CYLINDRICAL SHELL (HOOP)

- $I_x = \frac{m(6r^2+h^2)}{12}$
- $I_z = mr^2$



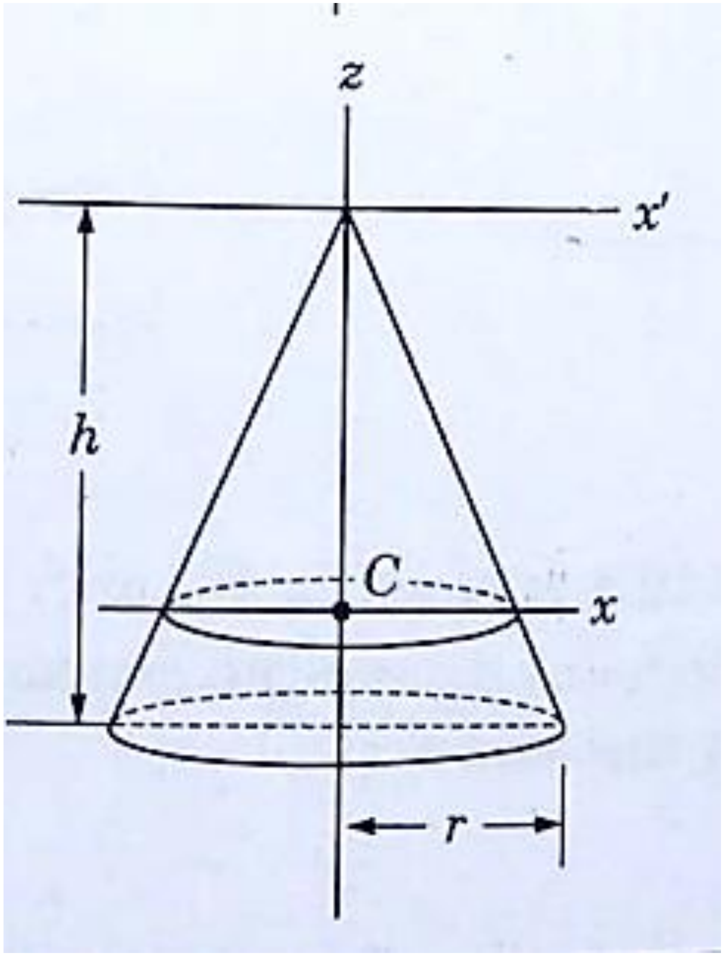
MOMENT OF INERTIA OF A MASS OF SPHERE

- $I_x = \frac{2}{5} m r^2$
- $I_z = \frac{2}{5} m r^2$



MOMENT OF INERTIA OF A MASS OF CONE

- $I_x = \frac{3}{80} m(4r^2 + h^2)$
- $I_x' = \frac{2}{5} m\left(\frac{r^3}{4} + h^2\right)$
- $I_z = \frac{3}{10} mr^2$



QUESTIONS: FILL IN THE BLANKS :

(*i*) The ...... is a rotational tendency of a force.

(*ii*) Moment of a force = ... × perpendicular distance.

(*iii*) Conventionally, clockwise moments are taken as ...... moments.

(*iv*) Sum of clockwise moments = sum of ...... moments.

(*v*) When coplanar forces meet in a point the system is known as ...... force system.

(*vi*) The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their ...... about that point.

(*vii*) Forces whose lines of action are parallel are called ...... forces.

(*viii*) Parallel forces are said to be like when they act in the ...... sense.

(*ix*) A ...... is a pair of two equal and opposite forces acting on a body in such a way that the lines of action of two forces are not in the same straight line.

(*x*) The moment of couple is known as ...... which is equal to one of the forces forming the couple multiplied by arm of the couple.

QUESTIONS: FILL IN THE BLANKS :

(*i*) The .. **moment**.... is a rotational tendency of a force.

(*ii*) Moment of a force = ... **force**... × perpendicular distance.

(*iii*) Conventionally, clockwise moments are taken as ... **negative**... moments.

(*iv*) Sum of clockwise moments = sum of .. **anti-clockwise**.... moments.

(*v*) When coplanar forces meet in a point the system is known as ... **coplanar concurrent**... force system.

(*vi*) The algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their .. **resultant**.... about that point.

(*vii*) Forces whose lines of action are parallel are called ... **parallel**... forces.

(*viii*) Parallel forces are said to be like when they act in the ... **same**... sense.

(*ix*) A ... **couple**... is a pair of two equal and opposite forces acting on a body in such a way that the lines of action of two forces are not in the same straight line.

(*x*) The moment of couple is known as ... **torque**... which is equal to one of the forces forming the couple multiplied by arm of the couple.